1. Introduction

1.1. Weigh-in-Motion

The development of technologies to weigh vehicles, and later individual axles, in motion started in the USA in the 1950s. Followed by fast developments, the Weigh-in-Motion (WIM) technology was available at elevated vehicle speeds in the 1970s. The versatility of the data acquired, which enabled, amongst others, the use of actual loads of heavy vehicles for pavement and bridge monitoring (e.g. O'Brien et al. 2013), pavement design and especially in law enforcement (excess weight control, fining), lead to a fast worldwide spread of WIM sites and the further development of the measuring technologies with regard to accuracy at high speeds, durability, filtering and quality assurance of the data acquired. Guidelines and specifications of WIM sites were assessed in the COST 323: Weigh-in-Motion of Road Vehicles action between 1992–1999, which aimed at unifying the technological aspects of the technology across Europe. The research covered requirements for WIM locations, accuracy requirements and classes as well as data assessment, making the findings of the report practically a European WIM standard. A part of the further research proposed by COST 323 was conducted in the European research WAVE (Weigh-in-Motion of Axles and Vehicles for Europe), involving algorithms to increase accuracy and data filtering, and the development of a new, carbon fibre weighing technology.

In 1996, with the financial support of the World Bank there were some 30 WIM sites installed in Hungary, which are still in use and data is continuously collected and stored, however only used for law enforcement purposes.

1.2. Hungarian pavement design

According to the e-UT 06.03.13 [UT 2-1.202:2005]: Aszfaltburkolatú útpályaszervezetek méretezése és mérése [Design of Road Pavement Structures and Overlay Design with Asphalt Surfacing] the standard Hungarian design traffic is calculated in passes of 100 kN equivalent single axle loads (ESALs). Load equivalency factors (LEFs) are used to calculate the cumulated damaging effect of various types of heavy vehicles compared to the standard axle load. The currently used factors were developed based on WIM data acquired during the first few years of operation between 1996–1997 (Csenki, Gulyás 1997), followed by minor updates. The factors were calculated based on one of the main findings of the AASHO road research, known as the Fourth Power Law, however the Hungarian standard shifted the fourth power to the fifth in the early 2000s. As the

Abstract. The load equivalency factors for pavement design currently in use by the Hungarian standard have been developed using Weigh-in-Motion data obtained during the first few years of operations after installing some 30 measuring sites in Hungary in 1996. In the past years, and currently, data is collected mainly at the border crossings of the country, however the data is used only for law enforcement purposes, and no comprehensive statistical analyses have been done. To develop actual load equivalency factors for the use in pavement design, data of one year was collected and statistical methods were applied. An algorithm was used to help managing the multimodal distribution of axle loads in mathematical perspectives. Monte-Carlo methods were applied to determine the factors for each heavy vehicle type and eventually for each vehicle class used by the current Hungarian pavement design manual. The calculated factors are considerably different from the current ones, indicating that the pavement design may lead to a false result. Furthermore, there are three vehicle types suggested to be incorporated into the standard due to their high occurrence.

Keywords: EM Algorithm, load equivalency factors, Monte-Carlo simulation, Weigh-in-Motion (WIM).
research showed the fifth power leads to less than 5% difference and the manual was updated (Gulyás 2002). Eventually, the calculation of the LEF for a given heavy vehicle is given in Eq (1):

\[
LEF_i = \sum_{j=1}^{k} \left( AF_j \cdot BN_j \cdot \frac{T_j}{T_e} \right)^h,
\]

where \( LEF \) = load equivalency factor; \( AF_j \) = correctional factor considering tyre type (1.00 for traditional twin tyre, 1.04 for super single, 1.06 for single tyre); \( BN_j \) = correctional factor considering tyre pressure (0.90–1.15 according to pressure value in kPa); \( T_j \) = axle load, kN; \( T_e \) = equivalent single axle load, 100 kN; \( h \) = exponent considering damage compared to the equivalent single axle (in Hungary \( h = 5 \)).

Analysis of the compliance of Eq (1) is out of the scope of the current paper. However previous research has shown that such factors as tyre pressure (Sivilevičius, Petkevičius 2002) and tyre type (Sebaaly, Tabatabaee 1992) have great influence on the submitted axle load to the pavement, as well as axle spacing within a given axle group (Gillmann 1999) and those factors should be considered. Despite their undisputable effects, partly due to their difficult measurability in a statistically convincing way, there are no relevant researches regarding proved data for such factors to be considered in this research, to the knowledge of the authors. It has also been observed that the effect of overloaded vehicles on the fatigue life of the pavement is severe due to the exponential effect (Coley et al. 2016; Pais et al. 2013; Zhao et al. 2012). The e-UT 06.03.13 considers the most frequently occurring heavy vehicle types and combinations sorted into detailed and merged vehicle classes as shown in Table 1. Load equivalency factors are given for both the detailed and merged vehicle classes.

The pavement design manual provides methods to calculate the design traffic based on the detailed and the merged vehicle classes as well as based on known axle loads or known axle passes. However, as the results of the regular Hungarian traffic surveys, officially published each year, quantifies heavy vehicles sorted into the only merged vehicle classes, the calculation of design traffic is only possible accordingly in most cases. Thus, due to the lack of data, the methods based on detailed vehicle classes, known axle loads or known axle passes is only used in a few cases where the detailed data is known is easily estimated.

Using the quantity of heavy vehicles in each merged vehicle class, considering the load equivalency factors ans other factors which are out of the scope of the current study, the design traffic is calculated for a given design period. As previous research has shown a slow but constant growth of axle loads in Hungary (Gulyás 2012), the correct determination and regular revision of the factors is important for the correct calculation of the design traffic, especially considering the synergistic effect of the simultaneous growth in traffic volume and axle loads.

The use of WIM measurements for this purpose is obvious, as such data offers the possibility to categorise heavy vehicles and to describe the whole axle load spectra at the same time. Accordingly, WIM data was used to inspect the factors regularly until 2010, however the values have been constant since 2005.

2. The acquired WIM data

The operation and data assessment of WIM sites in Hungary is under the authority of the Hungarian Roads Agency, which operates the public road network as well. The use of the sites is limited to law enforcement (i.e. overweight control, fining) for the past few years and as to the knowledge of the authors the last published statistical analysis of vehicle and axle loads was conducted in 2009 (Gulyás 2009), leaving the factor unchanged. The vast majority of the sites are mainly located at border crossings (mostly on both exit and entry sides). After the access of Hungary to the Schengen area the mandatory weighing at the Slovenian, Austrian and Slovakian borders was terminated. Therefore WIM data for the current study was only collected from sites at non-Schengen Hungarian borders, i.e. the Ukrainian, the Romanian, the Serbian and the Croatian borders. This filtering of data is assumed to ensure a real axle load spectra, which contains data about overweight vehicles and axles, as compared to the Schengen border data. The WIM data for 2014 was collected from the database.

2.1. Structure of WIM data

The system provides automatic axle coding at each weighing, which distinguishes tractors, trailers and semitrailers, tyre type (single, dual, road-friendly suspensions), axle groups (single, tandem, tridem) and axle spacings. Loads are measured for each passing axle. In general all vehicles are clearly classified into the classes presented in Table 1 and unreal axle loads, false axle codes, or other inconsistencies clearly identify measurement errors.

2.2. Vehicle classification

There were approximately 2.4 million vehicles measured at the four borders in 2014. Based on axle codes, after filtering false data, about 93% of data was classified into the

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Detailed vehicle class</th>
<th>Merged vehicle class</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt;7.5\ t )</td>
<td>C1</td>
<td>C</td>
</tr>
<tr>
<td>7.5\ t</td>
<td>C2</td>
<td>D</td>
</tr>
<tr>
<td>16\ t</td>
<td>D1</td>
<td>E</td>
</tr>
<tr>
<td>26\ t</td>
<td>D2</td>
<td></td>
</tr>
<tr>
<td>36\ t</td>
<td>E1</td>
<td></td>
</tr>
<tr>
<td>46\ t</td>
<td>E2</td>
<td></td>
</tr>
<tr>
<td>56\ t</td>
<td>E3</td>
<td></td>
</tr>
<tr>
<td>66\ t</td>
<td>E4</td>
<td></td>
</tr>
</tbody>
</table>
detailed and merged vehicle classes presented in Table 1 based on vehicle type. Analysis of the remaining 7% of the data showed there are three additional vehicle types unconsidered in the classification, although occurring frequently. Suggesting three new vehicle types (DX1, DX2, DX3) enabled the assessment of further 5%, leading to the overall 98.18% utilization of the data available. The distribution of measurements for vehicle types, detailed and merged vehicle classes, used for further analysis is shown in Table 2.

Although the vast majority of the data concerns international traffic and the distribution is not entirely valid to describe properties of heavy vehicles in the national traffic, the consideration to delete vehicle classes C22, D22, E31 and E32 is suggested, provided that the further measurements on the national road network will corroborate the findings.

Table 2. Distribution of measurements

<table>
<thead>
<tr>
<th>No. of measured vehicles</th>
<th>Occurrence</th>
<th>Vehicle type</th>
<th>Vehicle sign</th>
<th>Detailed vehicle class</th>
<th>Merged vehicle class</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>~0.00%</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>167 966</td>
<td>7.10%</td>
<td>C1</td>
<td>C1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 788</td>
<td>0.24%</td>
<td>C21</td>
<td>C2</td>
<td></td>
<td>C</td>
</tr>
<tr>
<td>28</td>
<td>~0.00%</td>
<td>C22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 434</td>
<td>0.57%</td>
<td>D11</td>
<td>D1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 733</td>
<td>0.54%</td>
<td>D12</td>
<td>D1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 933</td>
<td>0.46%</td>
<td>D21</td>
<td>D2</td>
<td></td>
<td>D</td>
</tr>
<tr>
<td>489</td>
<td>0.02%</td>
<td>D22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54 812</td>
<td>2.32%</td>
<td>DX1</td>
<td>DX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 744</td>
<td>2.57%</td>
<td>DX2</td>
<td>DX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 122</td>
<td>0.68%</td>
<td>DX3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 966</td>
<td>0.25%</td>
<td>E11</td>
<td>E1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>214 371</td>
<td>9.07%</td>
<td>E12</td>
<td>E2</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>1 797 771</td>
<td>76.04%</td>
<td>E2</td>
<td>E2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>667</td>
<td>0.03%</td>
<td>E31</td>
<td>E3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>~0.00%</td>
<td>E32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 452</td>
<td>0.10%</td>
<td>E4</td>
<td>E4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. Example of gross weight and axle load histograms for semi-trailers
It should be noted that as the measurements were made at border crossings, there is no convincing data on buses and coaches. This is mostly because such vehicles are relatively rare at border crossings and to a lesser extent the misqualification of bus axle codes is possible. Table 3 shows that the axle pattern according to vehicle type C1 is the same as for buses (vehicle type B).

3. Methodology

3.1. Distribution of axle loads

Figure 1 shows an example of gross weight and axle load spectra for the most frequently measured E2 semi-trailers. As seen, except for axle No. 1, load distributions have two or more peaks, indicating there are multiple load stages distinguished. As seen, the use of normal distributions, as shown in COST 334: Effects of Wide Single Tyres and Dual Tyres is inadequate to describe the load spectra. Instead, a multimodal distribution must be used. Experience showed that multimodality is true for all axles analysed and for all vehicle types.

The proper mathematical description using continuous functions of the axle load spectra will be essential for the proposed method presented in Section 3.4.

3.2. Mathematical representation of the data

Figure 1 shows that the axle load histograms have usually two or more peaks, therefore, specific distributions are inadequate to describe the spectra. The obvious solution is therefore to find a mixture of Gaussian distributions that fit the data. The probability density function of the Gaussian mixture distribution is described in Eq (2).

\[
p(x) = \sum_{k=0}^{K} \pi_k f \left( x \mid \mu_k, \sigma_k^2 \right).
\]  

(2)

where \( f \left( x \mid \mu_k, \sigma_k^2 \right) \) – a Gaussian probability density function with expected value of \( \mu_k \) and variance of \( \sigma_k^2 \), while for weights \( 0 \leq \pi_k \leq 1 \), \( \sum_{k=0}^{N} \pi_k = 1 \). The best Gaussian mixture distribution were determined using the EM Algorithm, followed by various fit tests to determine whether the actual data could be in fact derived from the theoretical estimation function. Results showed, in accordance with results shown in BASt 2009, Impact of Heavy Goods Traffic on the Bridges of Federal Highways, that the fit achieved using a mixture of three Gaussian distributions was adequate in all cases. Accordingly, the algorithm is presented hereinafter for \( K = 3 \).

The EM Algorithm is a method to iteratively calculate the maximum likelihood estimation (Dempster et al. 1977). In the first step (“E” step, Expectation), the proportion \( P_{i,k} \) of the individual samples is calculated for the given measured data \( x = (x_1, x_2, \ldots, x_n) \) and the initial parameter vector according to Eq (3).

\[
P_{i,k} = \frac{\pi_k f \left( x_i \mid \mu_k, \sigma_k^2 \right)}{\pi_1 f \left( x_i \mid \mu_1, \sigma_1^2 \right) + \pi_2 f \left( x_i \mid \mu_2, \sigma_2^2 \right) + \pi_3 f \left( x_i \mid \mu_3, \sigma_3^2 \right)}
\]  

(3)

Subsequently, in the second step (“M-step”, Maximization) the log-likelihood function is maximised according to Eq (4).

\[
\sum_{i=1}^{n} \ln \left[ \frac{\pi_1 f \left( x_i \mid \mu_1, \sigma_1^2 \right) + \pi_2 f \left( x_i \mid \mu_2, \sigma_2^2 \right) + \pi_3 f \left( x_i \mid \mu_3, \sigma_3^2 \right)}{\pi_1 f \left( x_i \mid \mu_1, \sigma_1^2 \right) + \pi_2 f \left( x_i \mid \mu_2, \sigma_2^2 \right) + \pi_3 f \left( x_i \mid \mu_3, \sigma_3^2 \right)} \right].
\]  

(4)

Thereafter, the derivation of Eq (4) by \( \mu_k \), and by \( \sigma_k^2 \), results in Eqs (5) and (6), respectively.

\[
P_k^{(new)} = \frac{\sum_{i=0}^{n} P_{i,k} x_i}{\sum_{i=0}^{n} P_{i,k}},
\]  

(5)

\[
\mu_k^{(new)} = \frac{\sum_{i=0}^{n} P_{i,k} \left( x_i - \mu_k \right)^2}{\sum_{i=0}^{n} P_{i,k}}.
\]  

(6)

Using the condition that \( \pi_1 + \pi_2 + \pi_3 = 1 \), \( \pi_k^{(new)} \) equals Eq (7), i.e. \( \theta \) parameter vector is updated to \( \theta^{(new)} \).

\[
\pi_k^{(new)} = \frac{\pi_k}{\sum_{i=0}^{n} P_{i,k}}.
\]  

(7)

The iteration repeatedly done several times results in a series of \( \theta \) parameters, which will eventually converge to the maximum likelihood estimation, giving the best fit for the mixture of three Gaussian distributions (Jordan, Lei 1996). Fig. 2 shows the fit of the distribution to the data after various iterations using the EM Algorithm.

As seen in Fig. 2, the Gaussian mixture distribution seems to fit after about 500 iterations. To determine the required number of iterations the fit tests were conducted as presented in Section 3.3.

3.3. Fit tests and results

The EM Algorithm iterates the parameters for the best fitting mix of specified number of Gaussian distributions. Regardless of this information the fit of the iterated (theoretical) Gaussian mixture distribution to the measured data is unsure. For example, Fig 3a shows the best fitted mixture of two Gaussians, and Fig. 3b shows the best fitted mixture of three Gaussians for the same axle load spectra.
To verify whether the measured data could be in fact derived from the iterated theoretical distributions determined using the algorithm, two types of fit tests were conducted. Both tests, the Anderson–Darling and the Cramér–von Mises, imply the analysis of the weighted distance between the empirical and theoretical distribution function according to Eq (8):

\[
d = \int_{-\infty}^{\infty} \left[ F_n(x) - F(x) \right]^2 w(x) dF(x),
\]

where \( d \) – difference between the theoretical function and the measured data; \( F_n(x) \) – empirical distribution function; \( F(x) \) – theoretical distribution function; \( w(x) \) – weight function.

The difference between the two tests lies in the weight function. In case of the Cramér-von Mises test \( w(x) \equiv 1 \), while in case of the Anderson-Darling test the weighting function is \( w(x) = F(x) \left[ 1 - F(x) \right]^{-1} \) (Stephens 1974).

Provided the measured values are in ascending order \( x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n \), test statistic \( S_{AD} \) is calculated according to Eq (9), and \( S_{CvM} \) according to Eq (10) in case of the Anderson-Darling and the Cramér-von Mises test, respectively:

\[
S_{AD} = -\sum_{i=1}^{n} \frac{2i-1}{n} \ln \left[ f(x_i) \right] - \ln \left[ 1 - f(x_{n+1-i}) \right] - n,
\]

\[
S_{CvM} = \frac{1}{12n} + \sum_{i=1}^{n} \left( \frac{2i-1}{n} - f(x_i) \right)^2,
\]

where \( f(x) \) – the theoretical distribution function. FFit test results showed that the mixes of three Gaussian distributions provide a fit to the measured data, i.e. the data is considered as a mixed Gaussian distribution, with the parameters calculated using the EM Algorithm. As the monitoring of heavy vehicle traffic parameters is crucial for a number of engineering problems, including pavement design as the aim of this paper, regular data analysis is required, which implies regular calculations on large amount of data. The experience with only one-year data showed that a high level of computer capacity is required, especially with regard to data storage. The presented
method also helps achieve efficient storage and computation of detailed data.

### 3.4. Determination of load equivalency factors

It is common to use WIM data to determine parameters for pavement design (Pais et al. 2013; Rys et al. 2016; Zofka et al. 2014). Previous research proved techniques to assess axle load spectra e.g. using regression analysis (Rys et al. 2015), nonparametric factorial analysis (Gao et al. 2005) or Bayesian model theory (Morales-Nápoles, Steenbergen 2014) or a robust approach involving simple distribution analysis (Savio et al. 2016). However there are difficulties with the use of such techniques in the everyday practice, and special abilities of the data analyst are required. The aim of the methodology presented in this paper is to provide a simple calculation method, yet to offer acceptable precision.

The LEF of a given vehicle type is calculated by the summation of the damaging effect compared to the standard axle for each axle. The proposed method to determine load equivalency factors, using the continuous mix distribution functions for each axle of each heavy vehicle type, based on Monte-Carlo simulation, is shown in Fig. 3.

The main steps to calculate the LEF for a given vehicle type are the following:

1) determination of the mixed Gaussian distribution parameters for each axle using the EM Algorithm;

2) random selection of axle loads with probability according to the distribution function;

3) calculation of LEF based on Eq (1) for each axle, and summation;

4) determination of the probability of occurrence for LEF calculated in step (3).

Performing the calculations sufficient times the result is the distribution function of the LEF for the given vehicle type – as experience has shown – as a normal distribution, which is easily managed for further calculations. Based on the distribution, the factors can be determined for the desired, e.g. as in engineering frequently used, 95% confidence level.

As presented in Section 1.2, according to the Hungarian pavement design guide the design traffic is calculated based on the merged vehicle classes. To obtain the required load equivalency factors for vehicle classes, the linear combination of calculated factors for individual vehicle types is required considering the proportion of the given vehicle types within a vehicle class, as shown in Fig. 4.

First equivalency factors for all vehicle types must be calculated based on the axle load spectra. Thereafter, considering the quantity of individual vehicle types, equivalency factors are calculated for the detailed vehicle classes. Using the same methodology the factors for the merged vehicle classes are calculated. Results, obtained using the proposed method, and factors according to the current standard are shown in Table 3.

As seen, the calculated factors for the most often used merged vehicle classes significantly differ from the values provided by the standard. LEF of “C” vehicle class is significantly lower than the standard value, probably because these are the most generally used commercial vehicles having relatively low operational costs (i.e. fuel and road pricing),
thus the implementation of high level logistics to minimise unloaded or partially loaded mileage in unimportant.

In contrast, LEF calculated for heavier vehicle classes than “C” are higher than the previous values provided by the standard, which shows that the level of logistics has improved, as operational costs of such vehicles is considerably higher (especially road pricing). In these cases it is often more economical to park such vehicles in wait for a payload than to operate idle. This fact confirms Fig. 1, showing that the gross weight of the vehicle shown is frequently close to its legal limit of 40 tonnes.

4. Conclusions
1. The multimodal distributions of heavy vehicle axle loads (and gross weights) are described with a continuous mathematical function of adequate precision for statistical

### Table 3. Current and calculated load equivalency factors for each vehicle type, the detailed and merged vehicle class

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Detailed vehicle class</th>
<th>Merged vehicle class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>No. of measured vehicles</td>
<td>Vehicle LEF</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>C1</td>
<td>167,966</td>
<td>0.30</td>
</tr>
<tr>
<td>C21</td>
<td>5788</td>
<td>1.51</td>
</tr>
<tr>
<td>C22</td>
<td>28</td>
<td>2.03</td>
</tr>
<tr>
<td>D11</td>
<td>13,434</td>
<td>1.31</td>
</tr>
<tr>
<td>D12</td>
<td>12,733</td>
<td>2.11</td>
</tr>
<tr>
<td>D21</td>
<td>10,933</td>
<td>2.23</td>
</tr>
<tr>
<td>D22</td>
<td>489</td>
<td>1.44</td>
</tr>
<tr>
<td>DX1</td>
<td>54,812</td>
<td>1.56</td>
</tr>
<tr>
<td>DX2</td>
<td>60,744</td>
<td>3.13</td>
</tr>
<tr>
<td>DX3</td>
<td>16,122</td>
<td>0.35</td>
</tr>
<tr>
<td>E11</td>
<td>5,966</td>
<td>0.59</td>
</tr>
<tr>
<td>E12</td>
<td>214,371</td>
<td>0.8</td>
</tr>
<tr>
<td>E21</td>
<td>1,797,771</td>
<td>1.94</td>
</tr>
<tr>
<td>E31</td>
<td>667</td>
<td>2.09</td>
</tr>
<tr>
<td>E32</td>
<td>70</td>
<td>0.27</td>
</tr>
<tr>
<td>E4</td>
<td>2452</td>
<td>1.49</td>
</tr>
</tbody>
</table>
analysis, using a mix of three Gaussian distributions. With regard to calculational and storage capacity needs of Weigh-in-Motion data this method is an efficient way to store and analyse several years of data as well.

2. The analysis of Weigh-in-Motion data shows that about 5–7% of heavy vehicles consist of single and biaxial trailers that are only indirectly considered in the current Hungarian standard. Incorporation of these vehicles and load equivalency factors is recommended during the pavement design.

3. Load equivalency factors for the detailed and merged vehicle classes, based on each vehicle type using the method presented have been calculated. As seen, the factors calculated from the recent data significantly differ from those provided in the current standard, therefore the update of the factors is recommended to refine the data used for pavement design, as well as regular inspection of data.

References


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