SIMPLIFIED ENGINEERING METHOD OF SUSPENSION BRIDGES WITH RIGID CABLES UNDER ACTION OF SYMMETRICAL AND ASYMMETRICAL LOADS

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Abstract. The basic disadvantage of suspension bridges can be considered their increased deformability. One of the ways to increase the rigidity of a suspension bridge is to transfer a part of stiffening girder rigidity to a suspension cable. A rigid cable better resists the imposed symmetrical and asymmetrical loads and displacements and retains its original geometric form. This paper presents a theoretical investigation of the suspension bridge with the rigid cable. Also, taking into account the geometrical non-linearity of rigid cable behaviour, a simplified analysis method for determination of forces and displacements in stiffening girder and cable of suspension bridge was developed.

Keywords: suspension bridge, rigid cable, stiffening girder, displacements.

1. Introduction

Suspension bridges possess a number of technical, economical and aesthetic advantages [1, 2]. The main problem of suspension bridges is excessive deformability, especially under actions of asymmetrical or local loadings [1–7]. Deformability of suspension systems depends on the kinematical character of displacements of a flexible suspension cable. The rigidity of suspension bridges is achieved by stiffening the bridge deck, i e increasing the height, and consequently the weight of a stiffening girder [5, 6]. The stiffness and stability of structures can be improved by inclined suspenders, a double-cable or combined systems. Reduction of kinematic displacements of the main cable can be achieved by a reduction of the sag-to-span ratio, but the smaller the sag of a cable, the greater are the cable forces and the required cross-sectional areas of the cables [4–6]. One of the ways to increase the rigidity of a suspension bridge is to transfer a part of stiffening girder rigidity to a suspension cable. By changing the ratio of the cable bending stiffness it is possible to reduce considerably the displacements of the suspension systems [5, 6]. Such an example is a suspension bridge in Pittsburgh [6]. Practically in this bridge there is no stiffness girder and all loads

are transferred to a rigid cable. The rigid cable successfully resists the imposed symmetrical and asymmetrical loads and displacements and retains its original geometric form [8]. The theoretical analysis of the influences of the main cables stiffness on the deformability of suspension bridge decks is presented in [9]. The main cables of increased stiffness make it possible to avoid other special stabilisation measures and local stresses, to increase resistance to corrosion; nodal and anchor connections become simpler. Analysis shows that there is a close relationship between the rigidity of the main cable, the character of applied load, and the displacements of the stiffening girder. It is possible to reduce deformability of suspension systems substantially by means of variation of the flexural stiffness of the cables [10].

It should be noted that there is a lack of scientific publications on theoretical analysis of suspension bridges with rigid cables. This paper presents a theoretical investigation of such a bridge. Taking into account the geometrical non-linear behaviour of a rigid cable, a simplified engineering method of analysis for determining forces and displacements in the stiffening girder and the cable of the suspension bridge was developed and presented in this paper.

2. Simplified engineering method for analysis of suspension bridges

The analysis of suspension bridges as systems of non-linear geometrical behaviour is complicated [1–5]. Therefore along with general analytical methods for analysing suspension bridges, simplified methods are used [6]. Adapting a beam analogy made it possible to obtain in this investigation simple and sufficiently accurate expressions for determination of forces and displacements for flexible cable and stiffening girder of a suspension bridge [6].

Creation of a simplified analysis method for suspension bridges with rigid cables under the action of symmetrical and asymmetrical loadings is certainly relevant. Two erection stages for such suspension bridges can be distinguished. In the first stage flexural stiffness is imposed on the cable from the very beginning of bridge erection, i e before it is loaded by the permanent and variable loads. In the second stage the cable attains flexural stiffness only when the bridge is loaded by the whole permanent load. Thus in the first case the rigid cable together with the stiffening girder carries both permanent and variable loads. In the second case the suspension element carries the whole permanent load in erection stage as an absolutely flexible cable, but the variable load is carried by the cable of finite rigidity and by the stiffening girder.

2.1. Design of suspension bridges with the rigid cable for the first stage of erection under the symmetrical loading

A suspension bridge subjected to the action of permanent q and variable p symmetrical loads is considered (Fig 1). In this case the part p_l of the total load is carried by the rigid cable and the other part p_s – by the stiffening girder:

$$q + p = p^* = p_l + p_s. \tag{1}$$

Due to the action of the part p_l the thrust force in the cable after its deformation with allowance for the cable flexural rigidity will be equal to:

$$H = \frac{p_l l^2}{8(f_0 + \Delta f)} - \frac{48 \Delta f EI_l}{5l^2}, \tag{2}$$

where Δf – deflection of the bridge at the middle of the span; f_0 – the initial sag of the cable.
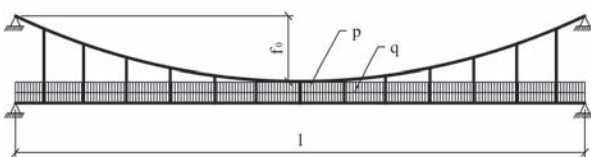

Fig 1. Symmetrically loaded suspension bridge

Under the action of the thrust force the cable elongates. This elastic elongation is defined by expression:

$$\Delta S = \frac{H \cdot l}{EA}. \tag{3}$$

For determining forces and displacements of the cable the equation of continuity of deformations is established:

$$S_1 = S_0 + \Delta S, \tag{4}$$

where $S_0 = l + \dfrac{8 f_0^2}{3l}$ – the initial cable length,

$S_1 = l + \dfrac{8(f_0 + \Delta f)^2}{3l}$ – cable length after deformation.

Thus, the equation (4) in an open form can be presented as:

$$l + \frac{8(f_0 + \Delta f)^2}{3l} = l + \frac{8 f_0^2}{3l} + \frac{H \cdot l}{EA},$$

or

$$\frac{16 f_0 \Delta f}{3l} + \frac{8 \Delta f^2}{3l} = \frac{H \cdot l}{EA}.$$

Ignoring the relatively small value Δf^2, the equation is obtained:

$$\frac{16 f_0 \Delta f}{3l} = \frac{Hl}{EA}. \tag{5}$$

Putting expression (2) for the thrust force of the cable into equation (5) and after rearranging the simplified formula for determination of deflection in the middle of the span of the bridge is obtained:

$$\Delta f \cong \frac{0{,}375 p_l l^4}{16 f_0^2 EA + 28{,}8 f_0 EI_l}. \tag{6}$$

There are two Δf and p_l unknowns in formula (6). According to [6] the part of the load taken by the cable will be equal to:

$$p_l = p^* - \frac{80 EI_s \Delta f}{l^4}. \tag{7}$$

Solving simultaneously (6) and (7), a direct solution for determining the deflection in the span middle of the bridge is obtained:

$$\Delta f \cong \frac{0{,}375 p^* l^4}{16 f_0^2 EA + 28{,}8 f_0 EI_l + 30 EI_s}. \tag{8}$$

When Δf value is determined, it is possible by the expression (7) to determine the part of the load taken by the cable, and using expression (1) – to determine the load part taken by the stiffening girder.

One of advantages of the presented method of analysis is that it is possible to determine directly deflection in the span middle and forces of a suspension bridge with rigid cables. In addition, using these expressions it is possible to solve a design problem, when the limit value of deflection Δf is known, both required rigidities of the cable EI_l and of the stiffening girder EI_s can be selected.

2.2. Design of suspension bridges with the rigid cable for the second stage of erection under the action of symmetrical loading

In this stage the whole permanent load q is carried by the cable, which is formed as absolutely flexible. The thrust force due to the permanent load for such a flexible cable is:

$$H_0 = \frac{ql^2}{8f_0}. \qquad (9)$$

When additional variable load is applied, a part of it will already be taken by the rigid cable, the thrust force value of which will be equal to:

$$H_1 = \frac{(q + p_l)l^2}{8(f_0 + \Delta f)} - \frac{48\Delta f EI_l}{5l^2}. \qquad (10)$$

The elastic elongation of the cable for this case is determined as follows:

$$\Delta S = \frac{(H_1 - H_0)l}{EA}. \qquad (11)$$

After solving the equation of continuity of deformations (4) expression for the second erection stage is obtained:

$$\Delta f \cong \frac{0,375(q + p_l)l^4 - 3l^2 H_0 f_0}{16 f_0^2 EA + 28,8 f_0 EI_l + 3l^2 H_0}. \qquad (12)$$

Taking into account (7), we obtain a direct solution:

$$\Delta f \cong \frac{0,375(q + p)l^4 - 3l^2 H_0 f_0}{16 f_0^2 EA + 28,8 f_0 EI_l + 30EI_s + 3l^2 H_0}. \qquad (13)$$

The main advantage of discussed engineering method for analysis of a suspension bridge is that possible stages of the bridge erection and loading history are evaluated by the method. It should be noted that when programs based on the finite element method are used, it is difficult to determine the cable forces and deflections and the stiffening girder for the second stage.

2.3. Design of suspension bridges with the rigid cable for the first stage of erection under the action of asymmetrical loading

A suspension bridge subjected to the action of symmetrical permanent q and asymmetrical variable p loads is considered (Fig 2).

Due to the action of a asymmetrical loading it is more convenient to design a suspension bridge by two stages. In the first stage the bridge span is symmetrically loaded by the permanent load q and a half of the variable load p:

$$p^* = q + \frac{p}{2}. \qquad (14)$$

When load p^* is known, it is possible to establish a deflection in the bridge span middle as in a case of symmetrical loading:

$$\Delta f \cong \frac{0,375 p^* l^4}{16 f_0^2 EA + 28,8 f_0 EI_l + 30EI_s}. \qquad (15)$$

A part of the load p^* taken by the cable will be equal to:

$$p_{1l} = p^* - \frac{80EI_s \Delta f}{l^4}. \qquad (16)$$

And a part of the load p^* taken by the stiffening girder will be equal to:

$$p_{1s} = \frac{80EI_s \Delta f}{l^4}. \qquad (17)$$

Accordingly, the thrust force in the cable after deformation will be equal to:

$$H = \frac{p^* l^2}{8(f_0 + \Delta f)} - \frac{48\Delta f EI_l}{5l^2}. \qquad (18)$$

In the second stage the suspension bridge span is loaded by the half of the variable load $p^{**} = \frac{p}{2}$ (Fig 3). In the left side of the bridge span direction of the load p^{**} is downwards, and in the right side – upwards. Considering that $c = \frac{l}{2}$, $d_1 = \frac{f + \Delta f}{4}$ and values of the deflection of mid-
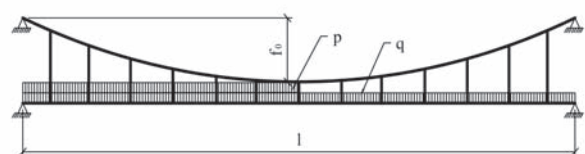

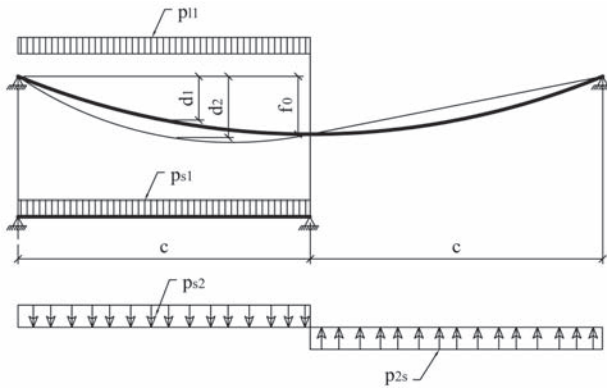**Fig 2.** Asymmetrically loaded suspension bridge

Fig 3. Suspension bridge scheme for the second design stage

dle of span Δf and cable thrust force H remain constant, it is simple to equal the displacement to the span quarters.

A part of the load p^{**} taken by the stiffening girder will be equal to:

$$p_{2s} = 0,5p - p_{2l}. \qquad (19)$$

The deflection of a quarter of bridge span has increased and became:

$$d_2 = \frac{(p_{1l} + p_{2l})c^2}{8H + \dfrac{384EI_l\Delta f}{5c^2}}. \qquad (20)$$

Difference between d_2 and d_1 is:

$$t = d_2 - d_1. \qquad (21)$$

The deflection of a quarter of bridge stiffening girder also is t:

$$t = \frac{5p_{2s}c^4}{384EI_s} = \frac{5(p - p_{2l})c^4}{384EI_s}. \qquad (22)$$

Accordingly:

$$t = d_2 - d_1 = \frac{(p_{1l} + p_{2l})c^2}{8H + \dfrac{384EI_l\Delta f}{5c^2}} - d_1 = \frac{5\times(0,5p - p_{2l})c^4}{384EI_s}. \qquad (23)$$

Solving the equation (23) with respect to p_{2l}, the part of the load p^{**} taken by the cable will be equal to:

$$p_{2l} = \frac{\dfrac{5\times 0,5pc^4}{384EI_s} + d_1 - \dfrac{p_{1l}c^2}{8H + \dfrac{384EI_l\Delta f}{5c^2}}}{\dfrac{5c^4}{384EI_s} + \dfrac{c^2}{8H + \dfrac{384EI_l\Delta f}{5c^2}}}. \qquad (24)$$

When p_{2l} value is determined, it is possible according to expression (19) to determine the part of the load taken by the stiffening girder and using the expression (21) – to determine the deflection of a quarter of bridge cable and stiffening girder.

2.4. Design of suspension bridges with the rigid cable for the second stage of erection under the action of asymmetrical loading

In this stage the whole permanent load q is carried by the cable, which is formed as absolutely flexible. In the first stage the suspension bridge span is loaded by a permanent load and a half of the variable load $p^* = q + \dfrac{p}{2}$. Deflection in the middle of bridge span will be equal to:

$$\Delta f = \frac{0,375p^*l^4 - 3l^2H_0f_0}{16f_0^2EA + 30f_0EI_l + 30EI_s + 3l^2H_0}, \qquad (25)$$

where $H_0 = \dfrac{ql^2}{8f_0}$ – the thrust force of the absolutely flexible cable.

When deflection Δf value is determined, it is possible according to expression (16) to determine a part of the load p^* taken by the cable, and according to expression (17) to determine a part of the load p^* taken by the stiffening girder. The thrust force in the cable after deformation will be equal according to the expression (18).

In the second stage the suspension bridge span is loaded by the half of the variable load $p^{**} = \dfrac{p}{2}$ (Fig 3).

According to expression (24) to determine a part of the load p^* taken by the cable and according to expression (21) – to determine the deflection of a quarter of bridge cable and stiffening girder.

3. Numerical example

The geometrical configuration of the model for the bridge considered here is shown in Fig 4.

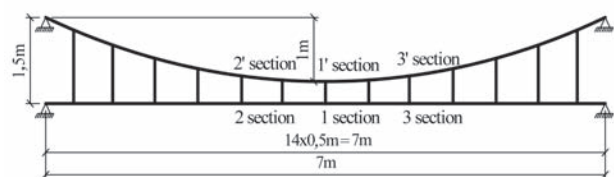The model for the bridge is a symmetrical reference structure, having a span of 7 m. The sag to span ratio of the

Fig 4. Suspension bridge model

Table 1. Cable to girder rigidities ratios and cross-sections

$\dfrac{EI_l}{EI_s}$	Section of stiffening girder, mm	Moment of inertia of stiffening girder, m⁴	Section of rigid cable, mm	Moment of inertia of rigid cable, m⁴
0,296	60×60×2	$2,8 \times 10^{-7}$	40×40×2	$8,5 \times 10^{-8}$
0,579	60×60×2	$2,8 \times 10^{-7}$	50×50×2	$1,7 \times 10^{-7}$
1,000	60×60×2	$2,8 \times 10^{-7}$	60×60×2	$2,8 \times 10^{-7}$

cable is 1:7. Distance between suspenders is 0,5 m. The model for the suspension bridge was analyzed under the action of symmetrical and asymmetrical loading. Finite element method (FEM) of analysis of the model for the bridge was performed by the program CosmosM.

For analytical analysis the simplified engineering method was applied with the allowance for stages of erection of the bridge structures.

Suspension rigid cable to girder rigidities ratios and cross-sections are presented in Table 1.

Symmetrical load $p = 3,0$ kN/m was assumed for calculation of stresses and displacements of the suspension bridge with rigid cables according to the engineering method for the first stage. For the second erection stage, the permanent symmetrical load $q = 2,0$ kN/m and variable one $p = 1,0$ kN/m were assumed.

Asymmetrical load was assumed taking into account variable and permanent loads ratio $\gamma = \dfrac{p}{q}$, varying it by $\gamma = 1$ to $\gamma = 3$. Permanent load $q = 1,0$ kN/m and variable load $p = 0,5$ kN/m, $p = 1,0$ kN/m, and $p = 1,5$ kN/m was assumed for calculating stresses and displacements according to the engineering method for the first stage of erection.

In the second erection stage the cable attains flexural stiffness only when the bridge is loaded by the whole permanent load $q = 1,0$ kN/m, the variable load $p = 0,5$ kN/m, $p = 1,0$ kN/m, and $p = 1,5$ kN/m is carried by the cable of finite rigidity and by the stiffening girder.

3.1. Results of numerical experiment performed by simplified engineering method and by FEM method for the first erection stage under a symmetrical load

Results of performed comparative analysis are presented in Tables 2 and 3. Results of calculation and of comparison are presented in Table 2 for the rigid cable and in Table 3 for the stiffening girder. It should be noted that according to the engineering method, values of stresses and deformations were determined for sections 2 and 2' (Fig 2), while according to the method of numerical simulation results are presented in Tables 2 and 3 for sections 1, 1', 2 ir 2'.

The peculiarity of the engineering method is that on the basis of equality of areas limited by deflection curves

Table 2. Comparison of displacements, forces and stresses for the rigid cable

Engineering method for the first stage of erection				
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴	H, kN	M, kN·m·10⁻³	σ, MPa
0,296	27	18,3	9,3	60
0,579	21	18,3	14	48
1,000	17	18,3	20	39

FEM, Non-linear							
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴		H, kN	M, kN·m·10⁻³		σ, MPa	
	1	2			1	2	1
0,296	35	28	18,3	11	10	65	63
0,579	28	23	18,3	18	17	52	50
1,000	24	20	18,3	25	23	44	42

Table 3. Comparison of displacements, stresses and forces for the stiffening girder

Engineering method for the first stage of erection			
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴	M, kN·m·10⁻³	σ, MPa
0,296	27	33	3,4
0,579	21	26	2,6
1,000	17	21	2,2

FEM, Non-linear						
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴		M, kN·m·10⁻³		σ, MPa	
	1	2	1	2	1	2
0,296	35	28	35	32	4,1	3,7
0,579	28	23	27	25	3,2	2,8
1,000	24	20	22	20	2,5	2,3

of the cable (quadratic parabola) and of the beam (fourth-order parabola), continuity conditions of displacements are satisfied in sections 2(2') and 3(3').

Errors for these sections, in comparison with results of numerical experiment, are less, moreover and for other sections the difference is not significant.

A diagram, indicating the cable variation and the stiffening girder displacements with the ratio of the cable to the girder flexural rigidities $\dfrac{EI_l}{EI_s}$ is presented in Fig 5. The diagram indicates that the errors of displacements values in sections 2 and 2' determined according to the engineering method do not exceed 15 %.

Diagrams indicating variations of bending moments in the cable and in the stiffening girder with the ratio of the cable to the girder flexural rigidities $\dfrac{EI_l}{EI_s}$ are presented in Figs 6 and 7.

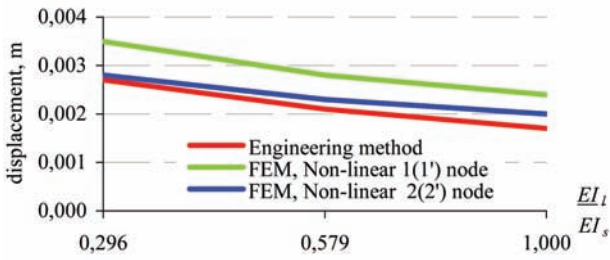The diagrams indicate that the bending moments in sections 2 and 2' in both the cable and the girder differ not

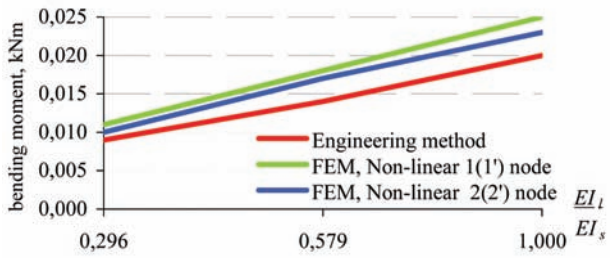**Fig 5.** Displacement of rigid cable and stiffening girder
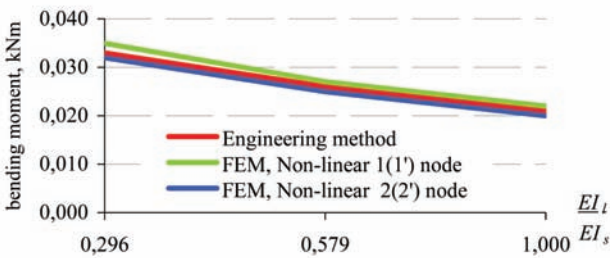

Fig 6. Rigid cable bending moments

Fig 7. Bending moments in stiffening girder

Table 4. Comparison of displacements, forces and stresses for the rigid cable

Engineering method for the first stage of erection				
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴	H, kN	M, kN·m·10⁻³	σ, MPa
0,296	27	18,3	9,3	60
0,579	21	18,3	14	48
1,000	17	18,3	20	39
Engineering method for the second stage of erection				
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴	H, kN	M, kN·m·10⁻³	σ, MPa
0,296	22	18,3	7,8	59
0,579	18	18,3	12	47
1,000	15	18,3	17	39

Table 5. Comparison of displacements, forces and stresses for stiffening girder

Engineering method for the first stage of erection			
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴	M, kN·m·10⁻³	σ, MPa
0,296	27	33	3,4
0,579	21	26	2,6
1,000	17	21	2,2
Engineering method for the second stage of erection			
$\dfrac{EI_l}{EI_s}$	Δf, m·10⁻⁴	M, kN·m·10⁻³	σ, MPa
0,296	22	27	2,9
0,579	18	21	2,3
1,000	15	18	1,9

much. Errors of bending moment values determined according to the engineering method do not exceed 10 %. On the ground of performed investigations, it is possible to state that the engineering method is sufficiently accurate.

3.2. Comparison of calculation results for the first and the second erection stages of the bridge under a symmetrical load

Calculation results of the first and second erection stages of the bridge using the engineering method are presented in Tables 4 and 5. Results of calculation of forces and displacements for the rigid cable are presented in Table 4 and for the stiffening girder – in Table 5.

Calculation results are presented for sections 1 and 1' (Fig 4). Tables 4 and 5 indicate displacements and forces determined by the engineering method in the second stage for both the cable and the girder are less. It can be explained by the fact that in the second erection stage the cable be-

haviour corresponds to the behaviour of an absolutely flexible cable. The cable carries the whole permanent load.

Diagram of variation of displacements for the cable and the stiffening girder determined by the engineering method with allowance for both erection stages in respect to the cable to the girder rigidity ratio $\dfrac{EI_l}{EI_s}$ is presented in Fig 8.
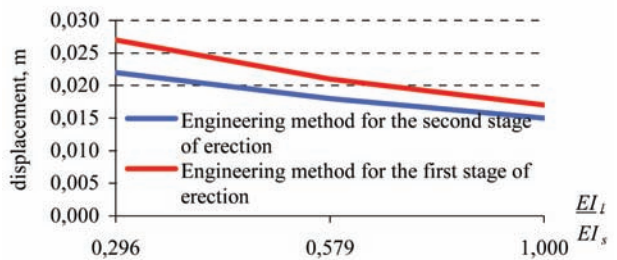
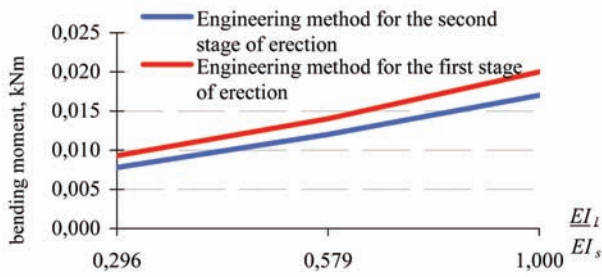**Fig 8.** Displacements of a rigid cable and a stiffening girder
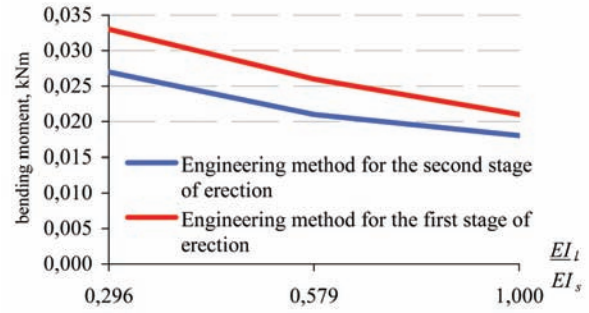
Fig 9. Rigid cable bending moments

Fig 10. Stiffening girder bending moments

It can be seen from the diagram that displacement values of the middle section of the cable and the girder determined according to the engineering method for the second erection stage are less about 18 %.

The diagram indicating variation of bending moments in the rigid cable with the cable to the girder rigidity ratio $\dfrac{EI_l}{EI_s}$ is presented in Fig 9. The diagram indicates that difference between erection stages is not great for smaller rigidity ratios. In the case of rigidity ratio $\dfrac{EI_l}{EI_s} = 1$ this difference is a bit greater.

Diagram indicating variation of bending moments in the stiffening girder with the cable to the girder rigidity ratio $\dfrac{EI_l}{EI_s}$ is presented in Fig 10. Similarly as with the rigid cable, difference between erection stages is not great for smaller rigidity ratios. The difference increases with rigidity ratio $\dfrac{EI_l}{EI_s}$.

3.3. Results of numerical experiment performed by simplified engineering method and by FEM method for the first erection stage under asymmetrical loads

Results of performed comparative analysis are presented in Tables 6 and 7. Results of calculation and of comparison are presented in Table 6 for the rigid cable and in Table 7 for the stiffening girder.

A diagram indicating the cable variation and the girder displacements stiffening with the ratio of the cable to the girder flexural rigidities $\dfrac{EI_l}{EI_s}$ is presented in Fig 11.

The diagram indicates that the errors of values of displacements o the left side of bridge span determined according to the engineering method do not exceed 18 % and on right side of the bridge span do not exceed 20 %.

Diagrams indicating variations of bending moments in the cable and in the stiffening girder with the ratio of the cable to the girder flexural rigidities $\dfrac{EI_l}{EI_s}$, then the ratio of variable and permanent loads intensities $\gamma = \dfrac{p}{q} = 1$ is presented in Fig 12.

Table 6. Comparison of displacements, forces and stresses for the rigid cable

$\dfrac{EI_l}{EI_s}$	Engineering method for the first stage of erection							FEM, Non-linear						
	t_k, m	t_d, m	H, kN	M_k, kN·m	M_d, kN·m	σ_k, MPa	σ_d, MPa	t_k, m	t_d, m	H, kN	M_k, kN·m	M_d, kN·m	σ_k, MPa	σ_d, MPa
	0,008	0,006	7,6	0,106	0,084	50	46	0,007	0,005	7,8	0,07	0,07	46	46
0,296	0,014	0,013	9,1	0,191	0,182	80	79	0,012	0,010	9,5	0,13	0,13	68	67
	0,021	0,020	11	0,279	0,268	109	111	0,017	0,016	11	0,19	0,20	90	93
	0,007	0,005	7,6	0,192	0,180	49	47	0,006	0,004	7,8	0,117	0,116	41	41
0,579	0,014	0,013	9,1	0,372	0,358	76	74	0,010	0,009	9,5	0,222	0,224	64	65
	0,020	0,019	11	0,544	0,527	103	100	0,013	0,015	11	0,326	0,342	85	88
	0,006	0,004	7,6	0,234	0,217	41	39	0,005	0,003	7,8	0,168	0,164	37	38
1,000	0,010	0,009	9,1	0,453	0,433	67	66	0,009	0,008	9,5	0,322	0,330	59	60
	0,014	0,013	11	0,661	0,637	92	91	0,012	0,010	11	0,470	0,491	79	82

Table 7. Comparison of displacements, forces and stresses for stiffening girder

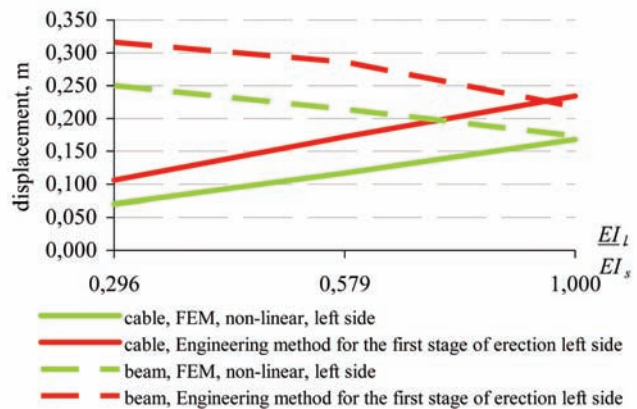| $\dfrac{EI_l}{EI_s}$	Engineering method for the first stage of erection						FEM, Non-linear					
	t_k, m	t_d, m	M_k, kN·m	M_d, kN·m	σ_k, MPa	σ_d, MPa	t_k, m	t_d, m	M_k, kN·m	M_d, kN·m	σ_k, MPa	σ_d, MPa
0,296	0,008	0,006	0,316	0,309	33	32	0,007	0,005	0,25	0,26	29	30
	0,014	0,013	0,610	0,602	63	62	0,012	0,010	0,48	0,51	56	59
	0,021	0,020	0,888	0,878	91	92	0,017	0,016	0,70	0,75	80	86
0,579	0,007	0,005	0,296	0,290	31	30	0,006	0,004	0,214	0,219	25	24
	0,014	0,013	0,570	0,564	59	58	0,010	0,009	0,408	0,433	47	49
	0,020	0,019	0,829	0,821	85	86	0,013	0,015	0,591	0,638	68	74
1,000	0,006	0,004	0,218	0,214	23	22	0,005	0,003	0,173	0,177	20	21
	0,010	0,009	0,416	0,411	43	42	0,009	0,008	0,331	0,353	38	41
	0,014	0,013	0,598	0,592	62	61	0,012	0,010	0,482	0,522	56	60
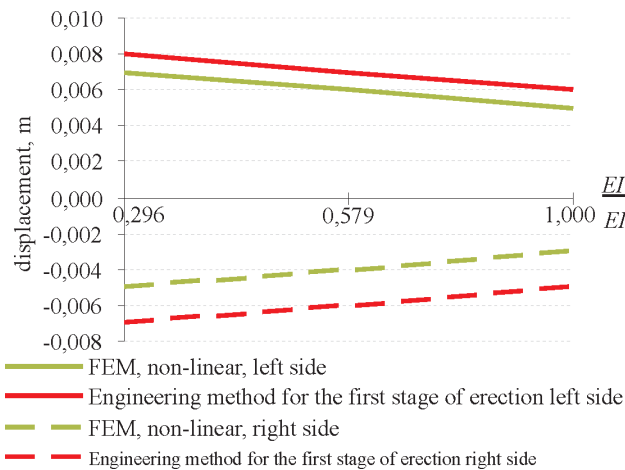
Fig 11. Displacements of rigid cable and stiffening girder the ratio of variable and permanent loads intensities γ = 1

Fig 12. Rigid cable and stiffening girder bending moments, the ratio of variable and permanent loads intensities γ = 1

The diagrams indicate that the bending moments in the left and right side of the bridge span in both the cable and the girder differ much. Errors of bending moment values determined according to the engineering method are about 29 %.

3.4. Comparison of results of calculation for the first and the second erection stages of the bridge under non-symmetrical load

Calculation results of the first and the second erection stages of the bridge under the action of the asymmetrical loading using the engineering method are presented in Tables 8 and 9. Results of calculation of forces and displacements for the rigid cable are presented in Table 8 and for the stiffening girder – in Table 9.

Diagram of variation of displacements for the cable and the stiffening girder determined by the engineering method with allowance for both erection stages in respect to the cable to the girder rigidity ratio $\dfrac{EI_l}{EI_s}$, then the ratio of variable and permanent loads intensities $\gamma = \dfrac{p}{q} = 1$ is presented in Fig 13.

It can be seen from the diagram that the displacement values of the left and right side of the cable and the girder determined according to the engineering method for the second erection stage are less about 13 %.
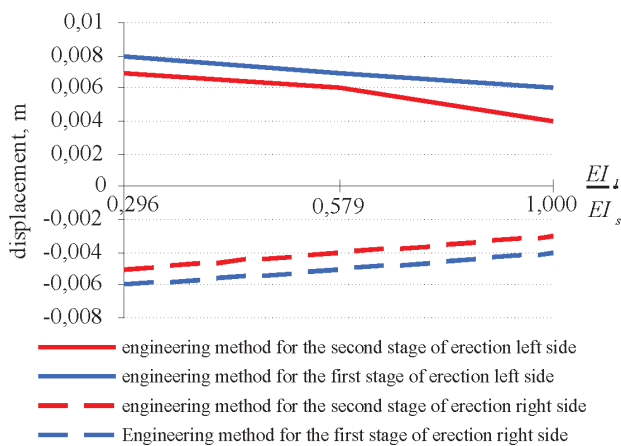
Diagram indicating variation of bending moments in the rigid cable with the cable to the girder rigidity ratio $\dfrac{EI_l}{EI_s}$ is presented in Fig 14.

Table 8. Comparison of displacements, forces and stresses for the rigid cable

$\dfrac{EI_l}{EI_s}$	Engineering method for the first stage of erection							Engineering method for the second stage of erection						
	t_k, m	t_d, m	H, kN	M_k, kN·m	M_d, kN·m	σ_k, MPa	σ_d, MPa	t_k, m	t_d, m	H, kN	M_k, kN·m	M_d, kN·m	σ_k, MPa	σ_d, MPa
	0,008	0,006	7,6	0,106	0,084	50	46	0,007	0,005	7,6	0,098	0,096	49	45
0,296	0,014	0,013	9,1	0,191	0,182	80	79	0,012	0,011	9,1	0,182	0,180	75	76
	0,021	0,020	11	0,279	0,268	109	111	0,020	0,019	11	0,268	0,263	105	100
	0,007	0,005	7,6	0,192	0,180	49	47	0,006	0,004	7,6	0,181	0,179	45	43
0,579	0,014	0,013	9,1	0,372	0,358	76	74	0,012	0,011	9,1	0,358	0,348	72	70
	0,020	0,019	11	0,544	0,527	103	100	0,018	0,017	11	0,528	0,526	99	97
	0,006	0,004	7,6	0,234	0,217	41	39	0,004	0,003	7,6	0,215	0,212	39	38
1,000	0,010	0,009	9,1	0,453	0,433	67	66	0,009	0,008	9,1	0,418	0,412	64	63
	0,014	0,013	11	0,661	0,637	92	91	0,013	0,012	11	0,611	0,601	88	86

Fig 13. Displacements of rigid cable and stiffening girder, the ratio of variable and permanent loads intensities
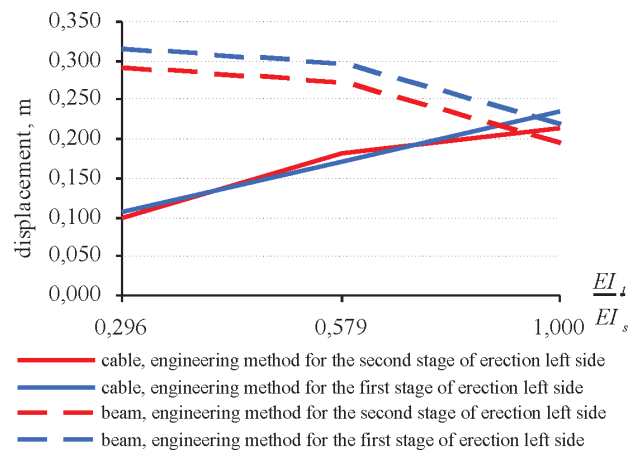

Fig 14. Displacements of rigid cable and stiffening girder, the ratio of variable and permanent loads intensities

The diagram indicates that that bending moment values of the left side of the cable and the girder determined according to the engineering method for the second erection stage are less about 8 %.

4. Conclusions

Application of cables of finite flexural rigidity for suspension bridges is an effective method for stabilization of their initial form. Design method presented in this paper gives an opportunity to take into account flexural rigidity EI_l of the cable under the action of symmetrical and asymmetrical loading. Two design cases are distinguished for evaluating two possible stages of erection of a suspension bridge.

Performed investigations and comparison of results of numerical experiment make it possible to state that the accuracy of the discussed engineering method is sufficient. The main advantage of this method for a suspension bridge analysis is that possible stages of erection and loading history of the bridge are evaluated in the method. Also the design solution can be readily obtained when the limit deflection Δf is known, flexural rigidities for both the cable EI_l and the girder EI_s can be selected.

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