



PROBABILISTIC ASSESSMENT OF SOIL SHEAR STRENGTH PARAMETERS USING TRIAXIAL TEST RESULTS

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Abstract. According to the currently valid standards, bearing resistance can be calculated applying direct informational-statistic method which includes probabilistic models of margin of resistance $Z = R - E$. Probabilistic methods allow to determine the influence of which design condition argument on the uncertainty of margin of bearing resistance is the greatest. That enables to project the directions of further investigations. The greatest influence on the uncertainty of margin of bearing resistance is made by the tangent of angle of internal friction and cohesion. Therefore it is required that these strength parameters should be determined more exactly. This would enable to achieve the economy of materials and labour expenditures for the construction of foundation without decreasing reliability of ground. In order to estimate the soil strength parameters more exactly, some changes in the triaxial test apparatus have been made so that the sample base has been allowed to move freely in the horizontal direction. Several experiments have been carried out by improved triaxial test apparatus. It has been determined that the values of the tangent of angle of internal friction and cohesion have been less by 10,8 % and 43 % respectively for dense sand samples with free horizontal base movement than those determined for samples with restricted horizontal movements of ends (regular ends). Foundation width B calculated according to the parameters of residual shearing strength, determined by usual triaxial test apparatus, is smaller by 23 % than that calculated according to the data obtained in the improved triaxial test apparatus. Reliability index of bearing resistance for a sample with regular ends calculated according to the residual values of soil shearing strength parameters by means of first order probabilistic methods for design approach 3 is $\beta = 4,4$.

Keywords: triaxial test, soil strength, probabilistic assessment, reliability index, design values, margin of resistance.

1. Introduction

Design of structure resistance is based on aiming at rational reliability of $P = 1 - \alpha$, taking into account constructional expenses and losses incurred during the period of service due to limit states. Designers should ensure safety, serviceability and durability of structure required by the standards. In recent years, the standards are based on partial factors method which is based on theoretical-experimental way of calculating methods for determining structure element resistance, strength etc. Errors of these models can be ascertained by means of comparing calculated resistance values with those determined by experiments. Therefore experiments and investigations ensuring the required structural element reliability have become particularly important. That is not irregular for errors to occur while designing, manufacturing, constructing and exploiting constructions. In addition, there is a possibility of consider-

able change in local conditions and construction quality. Thus, in order to ensure the required element reliability, it is essential that a control system be established for determining and eliminating the errors mentioned and undesirable quality deviations. For this purpose, verifying and certifying tests of structure elements, newly designed structures and constructions, as well as unique constructions under repair or reconstruction are carried out [1].

In partial factor method, design values of margins of resistance for the main variables are attached by partial safety and combination factors. Their determination is based either on the calibration of a long construction experience or on the statistical evaluation of experimental findings and construction observations. Eurocodes and Design Standard of Lithuania [2–3] provide information regarding structure resistance reliability evaluation methods (FORM first order reliability evaluation method, SECANT secant sec-

ond moments method). These methods allow calculations determining the design condition argument which makes the biggest influence on the uncertainty of margin of bearing resistance. Thus it is possible to foresee the directions of further investigations. The investigations carried out show that the biggest influence on the uncertainty of margin of bearing resistance is made by the tangent of angle of internal friction and cohesion; therefore it is essential that these strength parameters should be determined as precisely as possible. The experience of investigating soil mechanical properties reveal new shortcomings of testing apparatus. By eliminating these shortcomings, the apparatus for testing soil physical and mechanical properties as well as methods for processing test findings are improved, soil strength parameters are determined more exactly [4, 5].

Currently, triaxial and direct shear tests are used for determining soil shear strength parameters. Sandy soil strength parameters calculated by the triaxial test results are bigger than those obtained while calculating by means of the direct shear test [6].

It is assumed that soil sample deforms uniformly during the triaxial test. But it is not often the case that a sample in triaxial apparatus deforms uniformly during the test. Non-uniformity can be caused by the end restraint, sample height, insufficient drainage, membrane effects, soil compression, preparation of soil specimen, self weight [7–11]. The finite-element method analysis also shows that during triaxial testing distribution of stress and strain in the sample is non-uniform [12–18]. What is stress and strain distribution in soil sample, when a load is transmitted in a provided way? What influence does a non-uniformity have on the strength and stress-strain parameters of soil?

In order to determine what influence of apparatus pe-

culiarity makes on soil shear strength parameters, triaxial test apparatus have been improved, experiments have been carried out and influence of obtained results on bearing resistance have been estimated.

2. Experimental analysis

2.1. Identification of tested soil

During the experiment sand was tested. Type of soil according to Unified Soil Classification System is poorly-graded sand with fine SP–SM. Particles of sand are of rounded shape. The sand has an uniformity coefficient of 3,03, curvature coefficient 1,47, a specific gravity of soil particles was 2,67, maximum void ratio 0,745, minimum void ratio 0,502.

2.2. Influence of ends restraint on shear strength parameters

Determining shear strength parameters of soil according to triaxial compression test results is assumed, that, first, only normal stress on soil sample surface acts; second, soil sample deforms uniformly during the test. Experimental findings show that horizontal component of stress σ_x inside soil sample is distributed non-uniformly. Larger horizontal component of stress was found in the sides of soil specimen and the smaller one in the centre of specimen [17].

Investigating dense sands during standard triaxial compression testing, failure plane forms up and separate parts of the sample situated on the opposite sides of the plane are moving not only in vertical, but also in horizontal directions. Friction between the ends of the sample and plates is resisting to horizontal displacements. Not only normal but also tangential stress on the end of sample will act. Therefore for calculation of shear strength design parameters it should be evaluated. If horizontal displacement of sample ends are restricted, higher values of vertical stress will be required to move soil sample parts respectively each other in comparison with non restricted displacements of ends. In case of free movement of sample base, tangential stress at this end is eliminated.

Triaxial test apparatus have been improved for experiments: the friction between the sample ends and the plates was eliminated by allowing the sample base to move freely horizontally. In the geotechnical laboratory, triaxial tests with sand sample with ratio $H/D = 2$ have been carried out. Samples of 6 % water content have been prepared by compacting. Samples mass density was $\rho = 1,871 \text{ gr/cm}^3$ and void ratio of $e = 0,51$. The sample of one density under the same cell pressure has been sheared at least three times. Conditions of the sample boundaries: in the first case, when the sample top cap can rotate, and the friction between the sample ends and the plates is not eliminated (regular ends) (Fig 1a); in the second case, when the sample top cap can

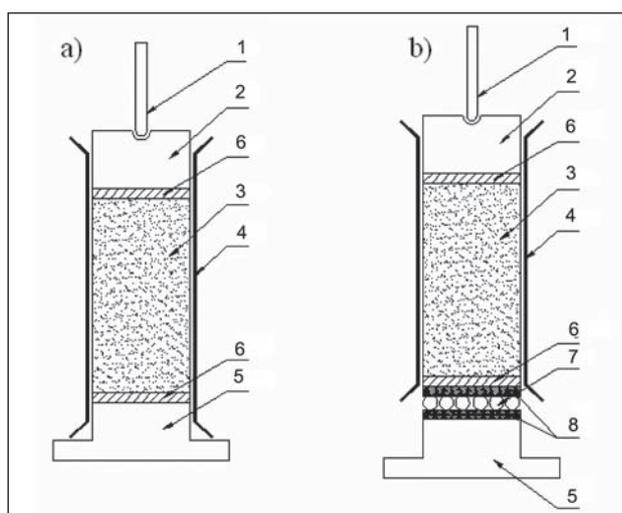


Fig 1. Triaxial test samples: a) with regular ends; b) with free horizontal movement of sample base: 1 – rod, 2 – cap, 3 – specimen, 4 – latex membrane, 5 – pedestal, 6 – porous stone, 7 – thrust bearing, 8 – stainless steel plates

rotate, and the friction between the sample base and plate is eliminated (free horizontal movement of sample base) (Fig 1b).

Comparison of test results obtained in triaxial apparatus for dense sand sample with free horizontal movement of base and for sample with regular ends (standard triaxial compression testing) shows that shape of graphs $\sigma_1 = f(\sigma_1 - \sigma_3)$ are different (Fig 2). Values of vertical component of stress vary evenly till it research relative axial strain $\epsilon_1 \approx 3\%$. In the first case of sample with regular ends, when the failure plane begins developing, vertical component of stress decreases evenly as the axial strain increases. In the second case of free horizontal movement of sample base, when the axial strain of 4–5% is achieved, a significant vertical component of stress decrease is observed; then the curve declines very insignificantly, it remains stable. In this case vertical component of stress in the bottom of sample is up to 10% smaller in comparison with vertical stress for a sample with regular ends, when relative axial deformation is equal to 15%.

Characteristic values of soil shear strength parameters were calculated by means of methods provided in design standard [19]. Characteristic values of the tangent of angle of internal friction obtained from maximum values of vertical stress σ_1 (peak angle of internal friction) in both cases differs insignificantly. Difference is not higher than 5%. Characteristic values of the tangent of angle of internal friction obtained from values of vertical component of stress, when relative axial deformation ϵ_1 is equal to 15% (residual angle of internal friction), for sample with a free horizontal movement of base are up to 10,8% smaller than for sample with regular ends (Fig 3).

Characteristic values of cohesion obtained from maximum values of vertical stress σ_1 (peak value of cohesion) in both cases differ in ~30% (Fig 4), when relative axial deformation ϵ_1 is equal to 15% (residual value of cohesion) differs in 43% (Fig 4).

3. Probabilistic evaluation of margin of bearing resistance

Structural Eurocodes [2] are now being prepared basing on the first order probabilistic methods. These methods are most generally used for partial factors calibration or the purposes of comparing construction reliability. Design Standard of Lithuanian Republic [3] provides three methods of design structure reliability. Direct informational-statistical (DIS) is one of them.

DIS designing may be carried out using SECANT secant second moments method [20]. This method is applied to design future members, for assesment of existing members of construction works and standard calibration. While applying theoretical model of this method, the uncertainties of all the arguments X_1, X_2, \dots, X_n that influence un-

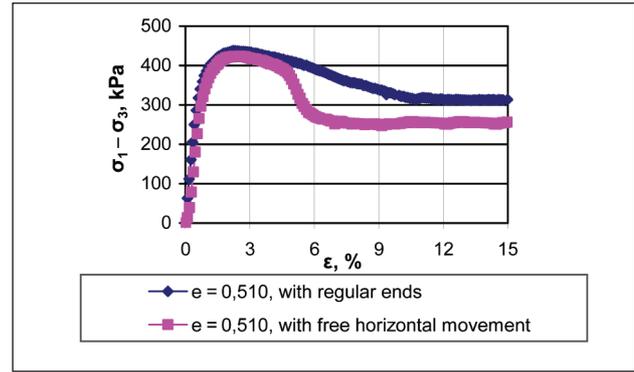


Fig 2. Stress-strain obtained in triaxial compression test on dense sand, when $H/D = 2, \sigma_3 = 100$ kPa

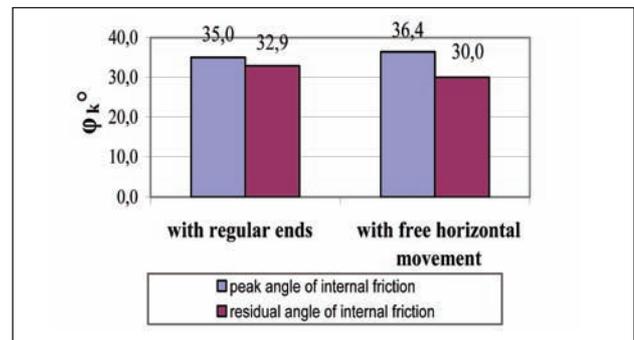


Fig 3. ϕ_k values from triaxial tests of sand specimens

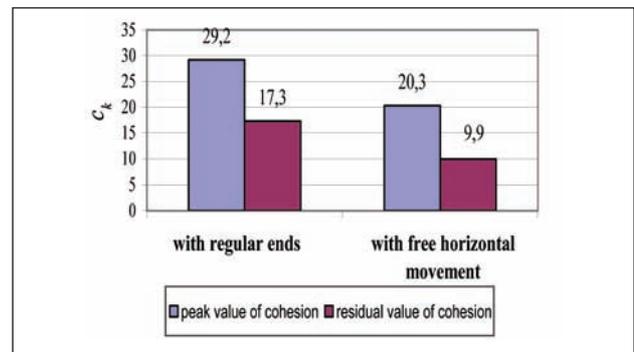


Fig 4. c_k values from triaxial tests of sand specimens

certainty of margin of resistance Z are taken into account. In addition, errors of models $\Delta R, \Delta Z$ for calculation of resistance R and effect of actions E may also be evaluated. Margin of resistance is expressed as:

$$Z = R - E + \Delta R + \Delta Z = z(X_1, X_2, \dots, X_n). \quad (1)$$

DIS method is based on random values functions Z according (1) to the mean μ_Z probabilistic assesment problem. Limit states probabilistic calculation is made by solving the equation system [20]:

$$z(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n} | A) - \beta \sigma_z = 0,$$

$$\frac{\Delta z(A)}{\Delta x_i} = [z(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n} | A) - z(\mu_{X_1}, \dots, \mu_{X_i} +$$

$$\beta \frac{\Delta z(A)}{\Delta x_i} \frac{\sigma_{X_i}^2}{\sigma_z}, \dots, \mu_{X_n} | A \Big] \beta \frac{\Delta z(A)}{\Delta x_i} \frac{\sigma_{X_i}^2}{\sigma_z},$$

$$i = 1, 2, \dots, n, \tag{2}$$

where

$$\sigma_z = \left[\sum_{i=1}^n \left(\frac{\Delta z(A)}{\Delta x_i} \sigma_{X_i} \right)^2 \right]^{0,5},$$

where μ_{X_i} and σ_{X_i} – the means and the standard deviations of normally distributed non-correlated random values $X_i, i = 1, 2, \dots, n$; if random random values X_i are distributed not according to the normal law, then they are appropriately normalised; β – reliability index, ie parameter of the Laplacian-Gaussian function $\Phi(\beta)$; A – parameter of equation (1), eg cross-section of structure element, foundation area etc are calculated in the course of designing.

In order to determine which design condition argument makes the biggest influence on the uncertainty of margin of resistance, the importance factor of argument should be calculated:

$$\alpha_{X_i}^2 = \left[\frac{\Delta z(A)}{\Delta x_i} \cdot \sigma_{X_i} \right]^2 / \sigma_z^2, \quad i = 1, 2, \dots, n. \tag{3}$$

DIS design may be carried out by applying statistical simulation (Monte Carlo) [21]. By means of this procedure, probability of the limit state of structure is calculated rather precisely. However, design values of margin of resistance and effect of actions arguments are not ascertained.

Design values of margin of resistance arguments can be calculated by solving optimisation problem according to the first order reliability method (FORM), which may be defined as failure surface $z_d = 0$ point located nearest to the distribution centre of the normalised argument space. It is calculated argument values of the design condition which would satisfy the design condition $z_d = 0$ itself and the probability function of these values would be at maximum:

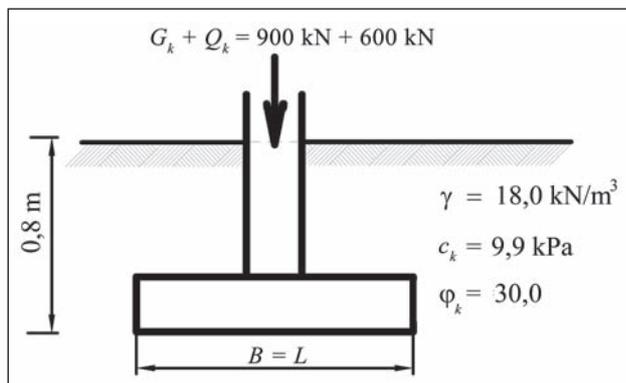


Fig 5. Diagram for design bearing resistance of spread foundation

$$f(x_1, x_2, \dots, x_n) \rightarrow \max,$$

$$z_d = g_i(x_{1d}, x_{2d}, \dots, x_{nd}) = 0, \tag{4}$$

where $f(x_1, x_2, \dots, x_n)$ – the probability density function of limit state arguments; $g_i(x_{1d}, x_{2d}, \dots, x_{nd})$ – design condition.

4. Probabilistic evaluation of bearing resistance calculated according to the results of triaxial test apparatus

Bearing resistance of spread foundation is calculated by means of methods provided in standard documents [19] using ground property parameters determined in the triaxial test apparatus according to the usual (regular ends) and improved methods (free horizontal movement of base) (Fig 5). Foundation measurements are calculated to satisfy the ultimate limit state:

$$E_d \leq R_d, \tag{5}$$

where E_d – design value of actions effect, R_d – design bearing resistance.

For drained conditions, the design bearing resistance R_d may be calculated from:

$$R_d = c' \cdot N_c \cdot s_c + q' \cdot N_q \cdot s_q + 0,5 \cdot \gamma' \cdot B' \cdot N_\gamma \cdot s_\gamma, \tag{6}$$

where c' – the effective cohesion, N_c, N_q, N_γ – dimensionless factors for bearing resistance, s_c, s_q, s_γ – dimensionless factors for the shape of the foundation; q' – the effective bulk weight density of soil at the foundation, base level, γ' – the effective bulk weight density of soil under foundation base, B' – the effective width of foundation.

The design value of actions effect is calculated as follows:

$$E_d = G + Q; \tag{7}$$

where G – permanent action, Q – variable action.

Foundation width B has been calculated according to the soil strength parameters determined by means of usual and improved triaxial test methods using peak and residual values of shear strength parameters (Fig 6). Foundation width calculated by usual method according to the peak values and the residual values of shear strength parameters are respectively 4 % and 23 % smaller than values calculated using an improved method.

Fig 7 obviously demonstrates that design bearing resistance R_d for design approach 3, calculated according to the residual values of shear strength parameters, determined by the improved test method, is 40 % smaller than that calculated using the residual values of strength parameters, determined according to the usual method. When R_d values are calculated according to the peak values of strength pa-

rameters determined by means of usual and improved method, the difference of R_d values obtained is 7 %.

Reliability index and argument values of design point G_{d^*} , Q_{d^*} , $\tan\phi_{d^*}$, c_{d^*} , B_{d^*} of bearing resistance designed according to [19] have been calculated using the FORM.

It is accepted that permanent action G , variable action Q , soil strength parameters $\tan\phi$, c and foundation width B are random values, whereas other arguments are known without deviations. Their values are given in Table 1. Standard deviations are established according to the published findings [22].

Figs 8, 9 demonstrate that values of arguments G_{d^*} , Q_{d^*} , $\tan\phi_{d^*}$, c_{d^*} , B_{d^*} at design point are different from design values G_d , Q_d , $\tan\phi_d$, c_d , B_d determined according to design standard. G_{d^*} , Q_{d^*} , c_{d^*} values are smaller, while $\tan\phi_{d^*}$, B_{d^*} values are obtained bigger than the design ones determined according to the design standard. Values G_{d^*} , Q_{d^*} , $\tan\phi_{d^*}$, c_{d^*} , B_{d^*} at design point satisfy the design condition, their probability density function at this point has the biggest value, while the value of reliability index obtained is the smallest.

Fig 10 demonstrates that reliability index b in tests with free horizontal movement of sample base is bigger than that of samples with regular ends. $\tan\phi$ of the latter samples is bigger, thus the obtained foundation width B and β are smaller. In samples with free horizontal movement of base, $\tan\phi$ decreases because friction between sample base and plates is evaluated, but B increases. Hence, foundation width calculated according to the results of usual triaxial tests is smaller than designed according to soil strength parameters determined in the improved triaxial test apparatus.

Importance factors of argument G , Q , $\tan\phi$, c , B have been calculated according to formula (3). These importance factors demonstrate arguments influence on the uncertainty of margin of bearing resistance. Uncertainty of margin of bearing resistance is mostly significantly influenced by the

Table 1. Probabilistic parameters of margin of bearing resistance Z arguments according to equation (1)

	Arguments of equation (1)				
	G	Q	$\tan\phi$	c	B
Mean values, μ_X	773	425	0,774	34,0	1,67
Standard deviations, σ_X	77,3	106	0,077	10,2	0,08
Characteristic values, X_k	900	600	0,647	17,3	1,53
Partial safety factors, γ_X	1,35	1,5	1,25	1,25	1,0
Design values, X_d	1 215	900	0,517	13,8	1,53
Argument values at design point, X_{d^*}	788	454	0,633	-6,0	1,63

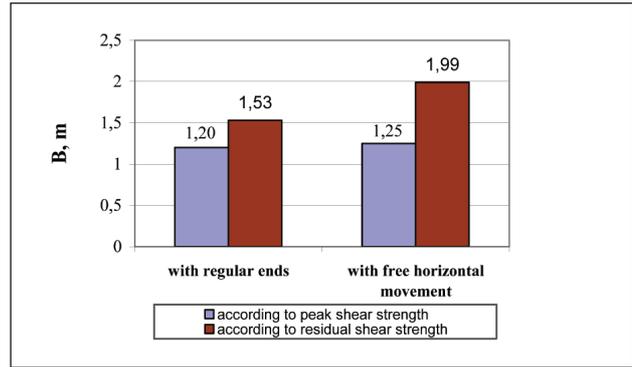


Fig 6. Values of foundation width B calculated according to peak and residual shear strengths

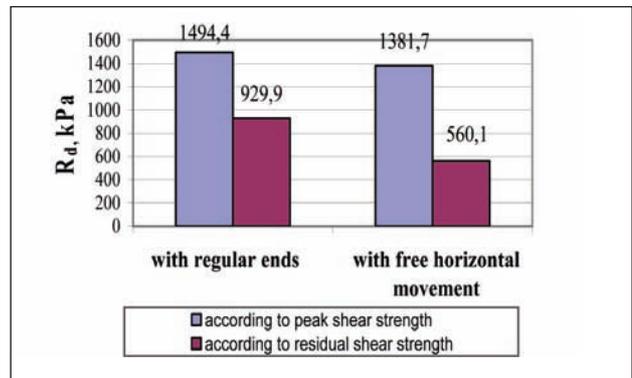


Fig 7. R_d values calculated according to design approach 3

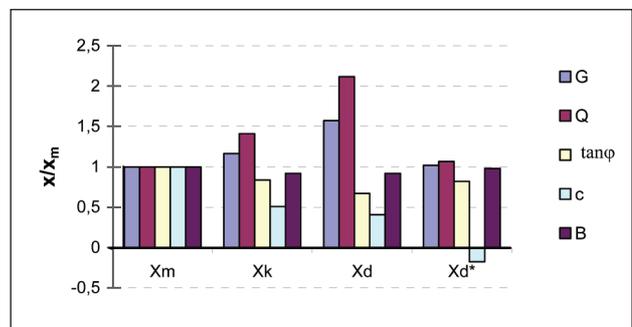


Fig 8. Values obtained for sample with regular ends using residual shear strength parameters

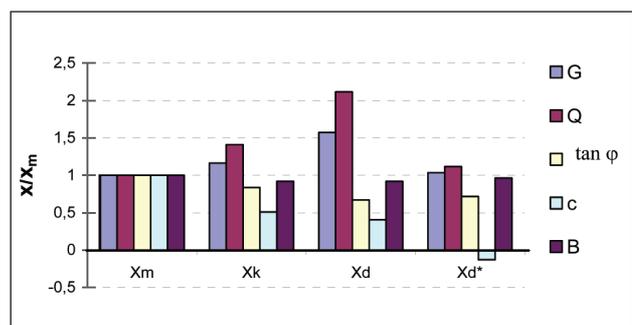


Fig 9. Values obtained for sample with free horizontal movement of base using residual shear strength parameters

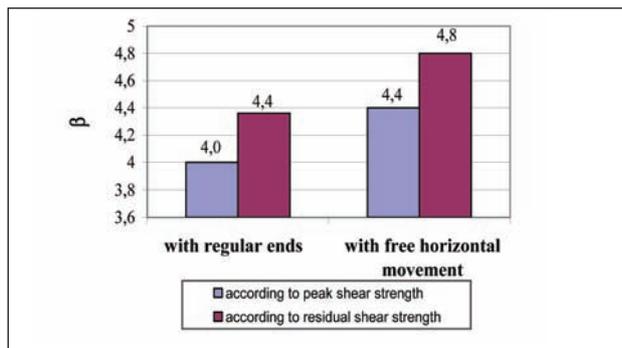


Fig 10. Values of reliability index β

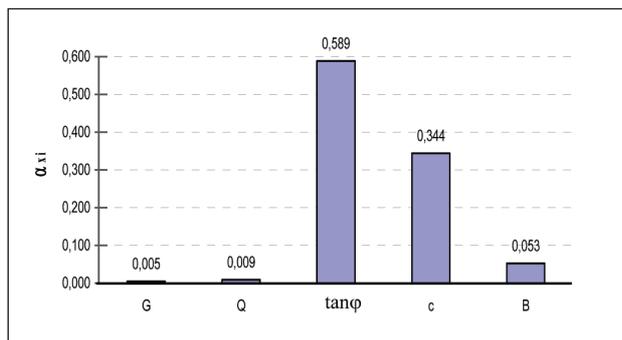


Fig 11. Influence of design condition argument on the uncertainty of the margin of bearing resistance

tangent of angle of internal friction and cohesion (Fig 11). Therefore it is essential that these shear strength parameters should be determined more precisely.

5. Conclusions

1. Comparison of test results obtained in triaxial apparatus for dense sand sample with free horizontal movement of base and for sample with regular ends shows that shape of graphs $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ are different.

2. Basing on the results obtained by testing soil in usual triaxial test apparatus, residual values of the angle of internal friction and cohesion have been calculated. They are respectively 10,8 % and 43 % bigger than those calculated according to the data obtained by soil test in improved triaxial test apparatus.

3. Foundation width B calculated according to the parameters of residual shearing strength, determined by usual triaxial test apparatus, is smaller by 23 % than that calculated according to the data obtained in the improved triaxial test apparatus.

4. The calculations made demonstrate that the biggest influence on the uncertainty of margin of bearing resistance is made by the tangent of the angle of internal friction and cohesion.

5. Design values of R_d calculated according to the residual values of soil shearing strength parameters deter-

mined in the usual triaxial test apparatus are by 40 % bigger than those calculated according to the residual values of soil shearing strength parameters determined in the improved triaxial test apparatus.

6. Reliability index of bearing resistance for sample with regular ends calculated by means of first order probabilistic methods for design approach 3 is $\beta = 4,4$.

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Received 16 May 2007; accepted 3 Aug 2007