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PRECAST SPUN CONCRETE PIERS IN ROAD BRIDGES AND FOOTBRIDGES

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Abstract. The usability of precast spun concrete members of annular cross-sections as pier shafts for road bridges and footbridges is discussed. The probability distribution of traffic loads is considered, their coefficient of variation is specified. First and second-order load effects for shafts of braced and bracing piers are analysed. The modeling of resisting compressive forces and bending moments of eccentrically loaded spun concrete shafts is considered. The features of mechanical properties of compressed spun concrete specimens reinforced by cold worked high-strength steel bars are presented. A simplified but fairly exact analysis of pier shafts under persistent situations by limit state and probability-based approaches is provided. A design of tubular shafts of braced piers using semi-probabilistic and probabilistic reliability verifications is illustrated by a numerical example.

Keywords: road bridges, spun concrete piers, high-strength steel, road traffic loads, second order effects, probability-based design.

1. Introduction

Precast spun (centrifugally cast) concrete shafts of annular cross-sections reinforced by steel bars uniformly distributed throughout their parameters and having high microcracking and low creep parameters satisfy economical, constructive and aesthetical requirements for bridges. The economically and structurally effective spun concrete shafts allow engineers to design and erect prefabricated piers subjected to vertical and horizontal forces. Therefore, they may be successfully used in construction practice of piers of short-span road bridges and footbridges. In some cases, it is expedient to use high-strength reinforcing steel bars increasing a bearing capacity of slender shafts exposed to compression with a small bending moment (Kudzys *et al.* 1993).

Multiple-span bridges may consist of one or few bracing piers and a large number of braced piers. The braced and bracing piers with tubular shafts may be treated as being isolated members fixed at their foundation which must also be fixed in the ground. The braced piers with movable bearings are almost free at their top and are assumed not to contribute to the overall horizontal stability of multiple-span bridges. The precast spun concrete shafts may be successfully used as bracing piers subjected to bending moments caused by vertical and horizontal forces. Therefore, the bracing piers should be fixed at their top.

The recommendations and directions presented in codes and standards for design and detailing rules of concrete structures *EN 1992-1:2004 Eurocode 2: Design of Concrete Structures – Part 1: General Rules and Rules for Buildings* and *ACI 318-99:1999 Building Code Requirements for Structural Concrete* are not fully formulated. In some cases, it hampers the development of analysis methods of spun concrete structures. Undoubtedly, the analysis of bearing capacity and structural safety of eccentrically loaded spun concrete piers under compression with a bending moment or bending with a compressive force has some characteristic features. Therefore, their design in a simple and easily perceptible manner is desirable by design engineers.

Contemporary design codes for bridge structures prescribe reliability verification methods exposed in limit state concepts. However, the reliability level of spun concrete piers designed by these concepts may differ considerably. The actual reliability level of piers may be defined only by probability-based concepts and approaches. However, for practical sake, the methodological and mathematical features of probabilistic approaches should be unsophisticated.

The object of this paper is stimulating the highway and structural engineers to use effective precast spun concrete shafts in bridge engineering and simplified but fairly exact semi-probabilistic and probabilistic approaches in their design practice.

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2. Destroying load effects

2.1. First order effects

The destroying compressive forces and bending moments of pier shafts are caused by permanent and transient loads of persistent situations. They may be grouped into vertical (gravity) and horizontal actions (Fig. 1b, c).

The permanent gravity forces

$$N_G = N_{G1} + N_{G2}$$

are caused by self-weight of structures, G_1, and roadway surfacing weight, G_2. The coefficients of variation of permanent loads are:

$$\delta G_1 \approx 0.10$$

and

$$\delta G_2 \approx 0.25$$

(Charnecki, Nowak 2008; Eamon, Nowak 2004). The value N_{G1} also depends on propping of precast members of continuous beams (Kudzys *et al.* 2007).

The variable gravity, N_Q, and horizontal braking, Q_l, live forces are caused by heavily-loaded trucks, cars and special vehicles. The forces Q_l may include temperature, concrete shrinkage and wind components. It is assumed that these horizontal forces act in longitudinal direction and are spread out over the entire pier cap of bracing piers.

The static, Q_{st}, and dynamic, Q_{din}, live loads of road girder bridges were investigated by Eamon and Nowak (2004) and Charnecki and Nowak (2008). It was determined that the coefficient of variation of static live loads, δQ_{st}, varies from 0.11 to 0.14–0.18 for a single and two heavily loaded trucks and the mean value of a dynamic factor may be taken as

$$\frac{Q_{din,m}}{Q_{st,m}} = 0.15 \text{ or } 0.10$$

with the coefficient of variation, δQ_{din}, equal to 0.80. The coefficient of variation of live loads and forces for road girder bridges may be expressed as:

$$\delta Q = \frac{\delta Q_{st} + \frac{Q_{din,m}}{Q_{st,m}} \delta Q_{din}}{1 + \frac{Q_{din,m}}{Q_{st,m}}}. \qquad (1)$$

For new bridges, the dynamic factor may be equal to 0.25 or 0.15. Thus, δQ by (1) can be introduced from 0.23 to 0.26 and may be taken equal to 0.25 or 0.30, when consequences of failure can be medium or high, respectively.

According to European Standards, the reliability verification of bridge structures is based on the limit state concept used in conjunction with partial factor methods. Using these design approaches, the multiplication factor, K_{F1}, should be applied to unfavourable actions using its value equal to 1.0 or 1.1, when consequences of failure may be medium or high, respectively, as it is recommended by *EN 1990:2002 Eurocode: Basis of Structural Design*.

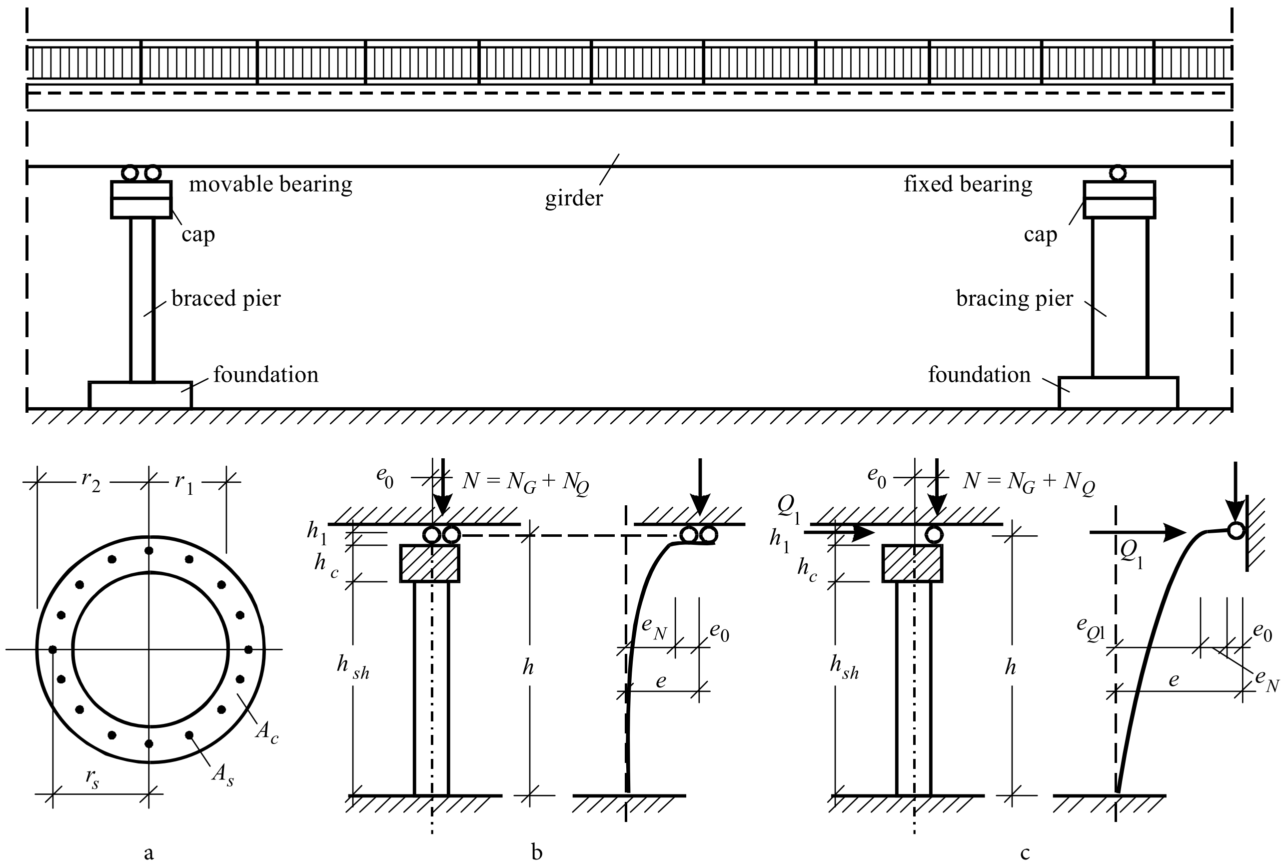

Fig. 1. Modelling of cross-sections (a) and eccentricities for spun concrete shafts of braced (b) and bracing (c) piers

According to *ENV 1991-3:1995 Eurocode 1 – Part 3. Basis of Design and Actions on Structures. Traffic Loads on Bridges*, the design values of load forces are equal to

$$N_{Ed} = 1.35(N_{Gk} + K_{F1}N_{Qk}),$$

$$Q_{ld} = 1.35K_{F1}Q_{lk},$$

where N_{Gk}, N_{Qk} and Q_{lk} are their characteristic values.

According to *EN 1992-2:2005 Eurocode 2: Design of Concrete Structures – Concrete Bridges – Design and Detailing Rules* recommendations, the flexural stiffness of pier shafts with constant cross-sections may be represented as follows:

$$EI = K_c E_c I, \quad (2)$$

where

$$K_c = \frac{0.3}{1 + 0.5\Phi \dfrac{M_{OG}}{M_{OE}}} \quad (3)$$

is the factor for effects of quasi-permanent loads, cracking and creep of concrete, where $\Phi = 1.2–2.0$ – the basic creep coefficient of concrete whose value depends on its strength class, dimensions of cross-sections of pier shafts and environmental conditions;

$$M_{OG} = N_G e_0 \text{ and } M_{OE} = Q_l h + N_E e_0$$

are the 1st order bending moments caused by permanent and total actions. For braced piers, the horizontal force

$$Q_l = 0$$

and

$$\frac{M_{OG}}{M_{OE}} = \frac{N_G}{N_E};$$

E_c – the modulus of elasticity of a concrete;

$$I = I_m = \frac{\pi\left(r_2^4 - r_1^4\right)}{4}$$

is the moment of inertia of a cross-section, where r_2 and r_1 – the radii of annular cross-section circles (Fig. 1a).

The mean value of the modulus of elasticity of a spun concrete may be defined by:

$$E_{cm} = 20(0.1 f_{cm})^{0.3}, \quad (4)$$

where f_{cm} – the mean value of concrete cylinder strength, MPa; *JCSS 2000 Probabilistic Model Code – Part 1: Basis of Design* suggests the coefficient of variation

$$\delta E_c = 0.15.$$

The coefficients of variation of cross-sectional area, A, and moment of inertia, I, of members may be determined by:

$$\delta A = \delta I = \frac{1.2 - 0.5\left(r_2 + r_1\right)}{150\left(r_2 - r_1\right)}. \quad (5)$$

According to *EN 1992-1:2004*, the design stiffness of shafts, $(EI)_d$, may be calculated from Eq (2), where

$$\frac{M_{OG}}{M_{OE}} \text{ and } E_c$$

are substituted by

$$\frac{M_{OGd}}{M_{OEd}} \text{ and } E_{cd} = \frac{E_{cm}}{1.2}.$$

The mean and variance of a flexural stiffness may be presented as:

$$(EI)_m \approx K_{cm} E_{cm} I_m, \quad (6)$$

$$\sigma^2(EI) = (E_{cm} I_m)^2 \sigma^2 K_c + (K_{cm} I_m)^2 \sigma^2 E_c + (K_{cm} E_{cm})^2 \sigma^2 I, \quad (7)$$

where the component statistics are:

$$K_{cm} = \frac{0.3}{1 + 0.5\Phi \dfrac{M_{OGm}}{M_{OEm}}}; \quad (8)$$

$$\sigma^2 K_c \approx \left[\frac{0.15\Phi M_{OGm}}{\left(M_{OEm} + 0.5\Phi M_{OGm}\right)^2}\right]^2 \times \left(\sigma^2 M_{OE} + \sigma^2 M_{OG}\right); \quad (9)$$

$$\sigma^2 E_c = (\delta E_c \times E_{cm})^2 = (0.15 E_{cm})^2; \quad (10)$$

$$\sigma^2 I = (\delta I \times I_m)^2 \text{ with } \delta I \text{ by Eq (5).} \quad (11)$$

According to *EN 1992-2:2005* directions, the 1st order eccentricity of a compressive force

$$N_E = N_G + N_Q$$

of piers (Fig. 1) may be given by:

$$e_0 = e_i + e_{sh}, \quad (12)$$

where

$$e_i = \frac{0.005h}{\sqrt{h}} \geq 0.00167h$$

and

$$e_i = \frac{h}{400}$$

are the inclination of in situ and precast piers, respectively, due to their geometrical imperfections;

$$e_{sh} = \frac{r_2}{15} \geq 20 \text{ mm}$$

is the shift of a bearing due to its movement or execution imperfections.

2.2. Second-order effects for braced piers

The buckling load of a shaft of braced piers may be expressed in terms of EI by Eq (2) and given by:

$$N_B = \frac{\pi^2 EI}{l_0^2}. \quad (13)$$

Its design value, mean an variance follow from:

$$N_{Bd} = \frac{\pi^2 K_{cd} E_{cd} I_m}{l_{0m}^2}, \quad (14)$$

$$N_{Bm} = \frac{\pi^2 K_{cm} E_{cm} I_m}{l_{0m}^2}\left(1 + 3\delta^2 l_0\right) \approx \frac{\pi^2 K_{cm} E_{cm} I_m}{l_{0m}^2}, \quad (15)$$

$$\sigma^2 N_B = \left(\frac{\pi^2 K_{cm} E_{cm} I_m}{l_{0m}^2}\right)^2 \sigma^2 E_c + \left(\frac{\pi^2 K_{cm} E_m}{l_{0m}^2}\right)^2 \sigma^2 I + \left(\frac{2\pi^2 K_{cm} E_{cm} I_m}{l_{0m}^3}\right)^2 \sigma^2 l_0 + \left(\frac{\pi^2 E_{cm} I_m}{l_{0m}^2}\right)^2 \sigma^2 K_c, \quad (16)$$

where the parameters E_{cm} by Eq (4),

$$E_{cd} = \frac{E_{cm}}{1.2},$$

K_{cm} – by Eq (8); $\sigma^2 K_c$ – by Eq (9); $\sigma^2 E_c$ – by Eq (10); $\sigma^2 I$ – by Eq (11);

$$l_{0m} = h$$

and

$$\sigma^2 l_0 = (\delta l \times h)^2 \approx (0.1h)^2.$$

Due to geometric imperfections, a buckling failure of a shaft under perfectly concentric compression is not a relevant limit state of piers. According to *EN 1992-1:2004*, a buckling load can be used as a parameter in their 2nd order analysis assuming that the 2nd order bending moment has a sine-shaped distribution. For braced piers (Fig. 1b), the 2nd order eccentricity of the applied compressive force N_E is defined as:

$$e = \frac{e_0\left[N_B + \left(\frac{\pi^2}{c_0} - 1\right)N_E\right]}{N_B - N_E}, \quad (17)$$

where N_B – by Eq (13); $c_0 = 8$ for a constant 1st order bending moment. The design value, mean and variance of a 2nd order eccentricity may be written as follows:

$$e_d = \frac{e_0\left[N_{Bd} + \left(\frac{\pi^2}{c_0} - 1\right)N_{Ed}\right]}{N_{Bd} - N_{Ed}}, \quad (18)$$

$$e_m = g\left(\xi_1,\ldots,\xi_n\right)_m + 0.5\sum\left(\frac{\partial^2 g}{\partial \xi_i^2}\right)_m \sigma^2 \xi_i \approx \frac{e_0\left[N_{Bm} + \left(\frac{\pi^2}{c_0} - 1\right)N_{Em}\right]}{N_{Bm} - N_{Em}}, \quad (19)$$

$$\sigma^2 e = \left[\frac{\frac{N_{Em} e_0 \pi^2}{c_0}}{\left(N_{Bm} - N_{Em}\right)^2}\right]^2 \sigma^2 N_B + \left[\frac{\frac{N_{Bm} e_0 \pi^2}{c_0}}{\left(N_{Bm} - N_{Em}\right)^2}\right]^2 \sigma^2 N_E, \quad (20)$$

where N_{Bd}, N_{Bm} and $\sigma^2 N_B$ – given by Eqs (14)–(16):

$$N_{Ed} = 1.35(N_{Gk} + K_{F1} N_{Qk}),$$

$$N_{Em} = N_{Gm} + N_{Qm},$$

$$\sigma^2 N_E = \sigma^2 N_G + \sigma^2 N_Q.$$

2.3. Second-order effects for bracing piers

The second-order eccentricity of the applied compressive force N (Fig. 1c) is of the form:

$$e = e_0 + e_{NG} + e_{NQ} + e_{Ql} = e_0 + \frac{e_0 h^2}{2EI}\left(N_G + N_Q\right) + \frac{Q_l h^3}{3EI}. \quad (21)$$

Its design value, mean and variance are as follows:

$$e_d = e_0 + \frac{e_0 h^2 N_{Ed}}{2(EI)_d} + \frac{Q_{ld} h^3}{3(EI)_d}, \quad (22)$$

$$e_m = g\left(\xi_1,\ldots,\xi_n\right)_m + 0.5\sum\left(\frac{\partial^2 g}{\partial \xi_i^2}\right)_m \sigma^2 \xi_i + \sum\sum\left(\frac{\partial^2 g}{\partial \xi_i \partial \xi_j}\right)_m \operatorname{cov}\left(\xi_i, \xi_j\right) \approx e_0 + \frac{e_0 h^2 N_{Em}}{2(EI)_m} + \frac{Q_{lm} h^3}{3(EI)_m}, \quad (23)$$

$$\sigma^2 e = \sigma^2 e_{NG} + \sigma^2 e_{NQ} + \sigma^2 e_{Ql} + 2\left(\frac{\partial e}{\partial N_Q}\right)_m \left(\frac{\partial e}{\partial Q_l}\right)_m \sigma N_Q \sigma Q_l. \quad (24)$$

For bracing piers, the total destroying moment and its design value are expressed as:

$$M_E = M_G + M_Q = N_G e + N_Q e + Q_l h, \quad (25)$$

$$M_{Ed} = N_{Gd}e_d + N_{Qd}e_d + Q_{ld}h. \quad (26)$$

where $N_{Gd} = 1.35N_{Gk}$, $N_{Qd} = 1.35K_{F1}N_{Qk}$, $Q_{ld} = 1.35K_{F1}Q_{lk}$.

3. Resisting compressive forces and bending moments

3.1. Compression with a bending moment

The compressive strength of spun concrete in bridge piers may have the form

$$f_{cc} = \alpha_{cc}k_2f_{ck}, \quad (27)$$

where the sustained load factor is expressed as:

$$\alpha_{cc} = 1 - 0.1\frac{N_P}{N_E} \text{ or } \alpha_{cc} = 1 - 0.1\frac{M_{OP}}{M_{OE}}, \quad (28)$$

and the strength factor is defined by:

$$k_2 = \frac{f_{c2}}{f_{ck}} = 0.85 - 1.7\rho, \quad (29)$$

where

$$\rho = \frac{A_s}{A_c}$$

is the reinforcement ratio (Fig. 1a); f_{ck} – characteristic cylinder strength, MPa. The coefficient of variation of this strength may be expressed by the Eq

$$\delta f_c = 0.088 + 3(70 - f_{ck})^2 \times 10^{-5}, \quad (30)$$

where f_{ck} is in MPa (Kudzys, Kliukas 2008).

The ultimate compressive stresses in reinforcing steel bars of concentrically and eccentrically loaded shafts of braced and bracing piers, respectively, may be defined as:

$$\sigma'_{sc} = 452(1.18 + 4\rho) \text{ (MPa)}, \quad (31)$$

$$\sigma_{sc} = 452(1.36 + 4\rho) \text{ (MPa)}. \quad (32)$$

According to test data, these values were not more as yield strength f_y and 800 MPa for hot rolled and cold worked steel bars, respectively, and were close to the stresses calculated by Hussaini *et al.* (1993) recommendations. The standardized second central moment of ultimate compressive stresses may be expressed as

$$\delta\sigma'_{sc} = \delta\sigma_{sc} = 0.105.$$

The shafts of braced piers, usually, are under compression with a small bending moment. The modelling of strain and stress distribution in concrete and high-strength steel of eccentrically loaded shafts may be based on a plane cross-section hypothesis and bi-linear concrete strain-stress relation when the conventional strain of concrete at its peak stress is equal to $0.5\varepsilon_{cu}$ (Fig. 2).

When the eccentricity ratio

$$\frac{e}{r_s} \le 1,$$

the response factors, characterizing an extent of the intelligent use of the compressive resistance of concrete and reinforcement cross-sections, may be calculated by the following Eqs:

$$k_c = \frac{N_{c1}(y_{c1} + r_s) - N_{c2}(y_{c2} + r_s)}{f_{cc}A_cr_s} \approx 1 - \frac{0.30e}{r_s(1 + 10\rho)}, \quad (33)$$

$$k_s = \frac{N_{sc}(y_{sc} + r_s) - N_{st}(r_s - y_{st})}{\sigma'_{sc}A_sr_s} \approx 1 - 0.34\frac{e}{r_s}, \quad (34)$$

where f_{cc} – given by Eq (27); σ'_{sc} – by Eq (31).

Using the response factors k_c and k_s, given by Eqs (33) and (34), the resisting compressive force N_R or the resistance R_N of eccentrically loaded shafts of annular cross-sections of braced piers may be presented in the form:

$$R_N = N_R = \frac{(k_cA_cf_{cc} + k_sA_s\sigma'_{sc})r_s}{e + r_s}, \quad (35)$$

where f_{cc} – given by Eq (27), σ'_{sc} – given by Eq (31), e – defined by Eq (17).

According to the partial safety factors design (PSFD), the design value of this resistance can be considered as:

$$R_d = N_{Rd} = \frac{(k_{cd}A_cf_{ccd} + k_{sd}A_s\sigma'_{scd})r_s}{e_d + r_s}, \quad (36)$$

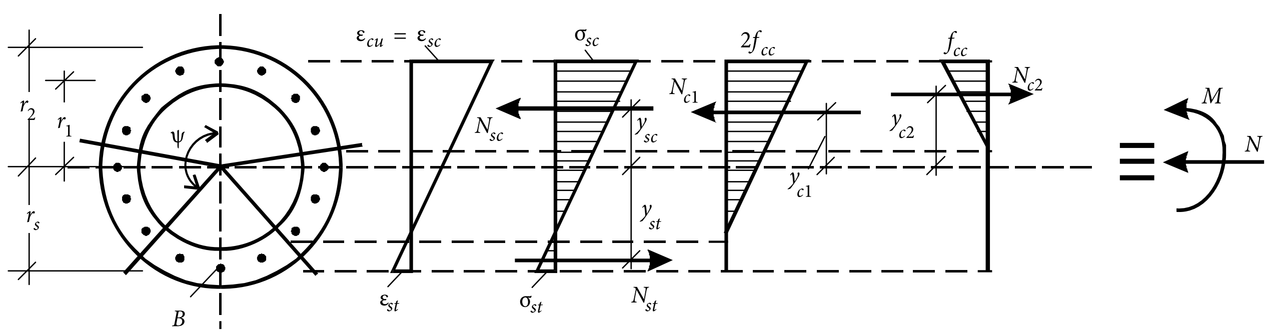

Fig. 2. The modelling of strain and stress distribution in the concrete and high-strength reinforcement of braced piers

where

$$k_{cd} = 1 - \frac{0.3e_d}{r_s\left(1+10\rho_m\right)};$$

$$k_{sd} = 1 - 0.34\frac{e_d}{r_s};$$

$$f_{ccd} = \frac{\alpha_{ccd}k_2 f_{ck}}{\gamma_c},$$

and

$$\sigma'_{scd} = \frac{\sigma'_{scm}}{\gamma_s}$$

with safety factors for materials

$$\gamma_c = 1.5$$

and

$$\gamma_s = 1.15$$

recommended by *EN 1992-1:2004*; e_d – given by Eq (18).

The statistics of resistance R_N may be presented in the forms:

$$R_{Nm} \approx \frac{\left(k_{cm}A_{cm}f_{ccm} + k_{sm}A_s\sigma'_{scm}\right)r_s}{e_m + r_s}, \quad (37)$$

$$\sigma^2 R_N = \left(\frac{\partial R}{\partial f_{cc}}\right)_m^2 \sigma^2 f_{cc} + \left(\frac{\partial R}{\partial A_c}\right)_m^2 \sigma^2 A_c + \left(\frac{\partial R}{\partial \sigma_{sc}}\right)_m^2 \sigma^2\sigma'_{sc} + \left(\frac{\partial R}{\partial e}\right)_m^2 \sigma^2 e + \left\{\left(\frac{\partial R}{\partial \rho}\right)_m^2 \sigma^2\rho + 2\left[\left(\frac{\partial R}{\partial A_c}\right)_m\left(\frac{\partial R}{\partial \rho}\right)_m \sigma A_c\sigma\rho + \left(\frac{\partial R}{\partial \sigma_{sc}}\right)_m\left(\frac{\partial R}{\partial \rho}\right)_m \sigma\sigma_{sc}\sigma\rho\right]\right\}, \quad (38)$$

where the variances of random variables of this resistance are:

$$\sigma^2 f_{cc} = (\delta f_{cc} \times f_{ccm})^2,$$

$$\sigma^2 A_c = (\delta A \times A_{cm})^2,$$

where δA – defined by Eq (5);

$$\sigma^2\sigma'_{sc} = \left(0.105\sigma'_{scm}\right)^2;$$

$\sigma^2 e$ – given by Eq (20),

$$\sigma^2\rho = \left(\frac{A_s}{A_{cm}^2}\right)^2 \sigma^2 A_c.$$

The standardized 2nd central moment of concrete strength in compression zones of shafts may be defined as:

$$\delta f_{cc} = \left(\delta^2 f_{c1} + \delta^2 f_{c2}\right)^{\frac{1}{2}} = 0.145,$$

where its components

$$\delta f_{c1} = \delta\sigma'_{sc} \approx 0.105$$

and

$$\delta f_{c2} = 0.08 - 0.12 \approx 0.10$$

as the coefficients of variation define the ultimate deformations of a spun concrete and the error of its bi-linear stress-strain relation. The values presented in braces of Eq (38) may be omitted if the approx coefficient of variation

$$\delta f_{cc} = 0.16$$

is applied.

3.2. Bending with a compressive force

According to Vadluga (Вадлуга 1985), the ultimate bending moment M_R of shafts of annular cross-sections (Fig. 3) reinforced by hot-rolled steel bars could be expressed as follows:

$$M_R = 1.2r_s\left(A_s f_{st} + N\right)\left(1 - \frac{\psi}{\pi}\right) = 1.2r_s\left(A_s f_{st} + N\right)\left\{1 - \frac{A_s f_{st} + N}{A_c f_{cc} + A_s\left(f_{st} + f_{sc}\right)}\right\}. \quad (39)$$

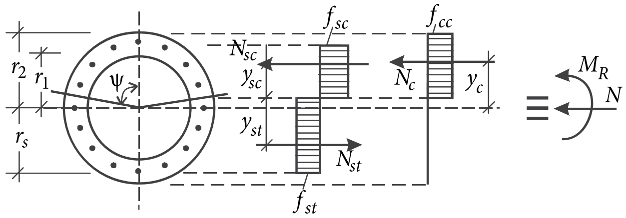

Fig. 3. The modelling of stress distribution in the reinforcement and concrete of bracing piers

The characteristic strength of reinforcement in tension, f_{stk}, and compression, f_{sck}, should be not more as f_{yk} and 500 MPa, respectively. The compressive strength of concrete, f_{cc}, is given by Eq (27). When a high-strength cold worked steel is used, its mechanical parameters may be defined as:

$$f_{stm} = 500 \text{ MPa},$$

$$f_{stk} = 0.9f_{stm} = 450 \text{ MPa},$$

$$f_{scm} = 600 \text{ MPa},$$

$$f_{sck} = 0.9f_{scm} = 540 \text{ MPa}.$$

For design practice, Eq (39) may be rewritten in the form:

$$M_R = \frac{T_2 T_3}{T_1}, \quad (40)$$

where

$$T_1 = A_c f_{cc} + A_s(f_{st} + f_{sc}); \quad (41)$$

$$T_2 = 1.2r_s(A_s f_{st} + N); \quad (42)$$

$$T_3 = A_c f_{cc} + A_s f_{sc} - N. \tag{43}$$

The design value of resisting moment, M_{Rd}, may be calculated by Eq (40) using the design values of strength of materials

$$f_{ccd} = \frac{k_2 f_{cck}}{1.5};$$

$$f_{std} = f_{scd} = \frac{f_{yk}}{1.15} \leq 435 \text{ MPa}$$

for hot-rolled steel bars and

$$f_{std} = \frac{f_{stk}}{1.15} = 390 \text{ MPa},$$

$$f_{scd} = \frac{f_{sck}}{1.15} = 470 \text{ MPa}$$

for cold worked reinforcement.

The statistics of resistance $R_M = M_R$ of pier shafts under bending with concentrical compressive force may have the forms:

$$R_{Mn} \approx \frac{T_{2m} T_{3m}}{T_{1m}}, \tag{44}$$

with the parameters

$$f_{ccm} = \alpha_{ccm} k_{2m} f_{cm},$$

$$f_{stm} \text{ and } f_{scm},$$

$$\sigma^2 R_M = \left[\frac{T_{2m}\left(T_{1m} - T_{3m}\right)}{T_{1m}^2}\right]^2 (A_{cm}^2 \sigma^2 f_{cc} + f_{ccm}^2 \sigma^2 A_c + A_s^2 \sigma^2 f_{sc}) + \left[\frac{A_s T_{3m}\left(1.2 r_s T_{1m} - T_{2m}\right)}{T_{1m}^2}\right]^2 \times \sigma^2 f_{st} + \left[\frac{1.2 r_s T_{3m} - T_{2m}}{T_{1m}}\right]^2 \sigma^2 N_E \tag{45}$$

with the following variances of variables:

$$\sigma^2 f_{cc} = (\delta f_c \times f_{ccm})^2,$$

$$\sigma^2 A_c = (\delta A_c \times A_{cm})^2,$$

$$\sigma^2 f_{sc} = (\delta f_s \times f_{scm})^2,$$

$$\sigma^2 f_{st} = (\delta f_s \times f_{stm})^2,$$

$$\sigma^2 N_E = \sigma^2 N_G + \sigma^2 N_Q.$$

The coefficients of variation of steel strengths f_{st} and f_{sc} may be modelled as:

$$\delta f_s = \left(\delta^2 f_{s1} + \delta^2 f_{s2}\right)^{\frac{1}{2}} \approx 0.10,$$

where the components

$$\delta f_{s1} \approx 0.06$$

and

$$\delta f_{s2} \approx 0.08$$

define their statistical uncertainties and the errors of right-angled epures.

The indispensable reliability level may not be achieved for strongly reinforced members under compression (Holicky, Markova 2007). The appropriate level of structural quality and reliability of piers and other members of bridges may be obtained only using probabilistic design approaches.

4. Probability-based design

4.1. Reliability index and model uncertainties

The generalized reliability index as a standard reliability measure of bridge members may be defined as:

$$\beta = \Phi^{-1} \mathbf{P}_s, \tag{46}$$

where $\mathbf{P}_s$ – the survival probability of a member and $\Phi^{-1}(\bullet)$ – the inverse Gaussian distribution. For columns designed by limit state approaches, this index is between 3.3 and 5.0. Its value increases significantly when a reinforcement ratio increases and a compressive strength of concrete decreases (Diniz 2005).

According to *EN 1990:2002*, the target reliability index, β_T, of structural members may be selected from 3.3 to 4.3 depending on their failure consequence classes. For eccentrically loaded reinforced concrete columns (beam-columns) and piers, the index β_T must be not less than 3.5. The failure of a pier can be more brittle compared to the failure of bridge span members and have influence on their structural safety. Therefore, this index may be selected equal to 4.0 (Nowak, Szerszen 2003; Szerszen, Nowak 2003; Szerszen *et al.* 2005). This value corresponds to the survival probability of pier shafts, $\mathbf{P}_s$, equal to 0.999968.

Usually both the resistance of a pier shaft and its action effects may be treated as stationary random processes. The safety margin of shafts of braced and bracing piers may be defined, respectively, as their performances of the forms:

$$Z_N(t) = g[\theta_N, \mathbf{X}_N(t)] = \theta_{RN} R_N - \theta_N N_G - \theta_N N_Q(t), \tag{47}$$

$$Z_M(t) = g[\theta_M, \mathbf{X}_M(t)] = \theta_{RM} R_M - \theta_M M_G - \theta_M M_Q(t), \tag{48}$$

where the components of the vector θ of additional random variables, as professional factors, represent the uncertainties of design models in transformation of the vector $\mathbf{X}(t)$ of variables into resisting and destroying action effects (Melchers 1999).

According to Ellingwood (1981), Melchers (1999), *ENV 1991-3:1995*, *EN 1990:2002*, *JCSS 2000* and *ISO 2394:1998 General Principles on Reliability for Structures,* the probability distributions of concrete shaft resistance R_N

or R_M and permanent action effect N_G or M_G are close to a normal distribution.

An application of a lognormal distribution is conventional for loads due to the road traffic consisting of the sum of a number of identically distributed independent lorries, cars and special vehicles. A stationary lognormal process is used by Bhattacharya (2008) in his investigations of bridge extreme loads. Thus, the probability distribution of live load effects may be treated as a lognormal one as it is recommended by *ISO 2394:1998*, Eamon and Nowak (2004). According to *JCSS 2000*, Holicky and Markova (2007), the means and standard deviations of the uncertainties of action effects for columns may be defined as:

$$\theta_{Nm} = \theta_{Mm} \approx 1.0,$$

$$\sigma\theta_N = \sigma\theta_M \approx 0.1.$$

The additional uncertainties of shaft resistances, representing the ratio between their actual and predicted values, may be expressed by its mean, θ_{Rm}, and standard deviation, $\sigma\theta_R$. According to test data determined by Vadluga (Вадлуга 1985), Kudzys and Kliukas (2008), these statistics may be defined as:

$$\theta_{RNm} = 0.99, \sigma\theta_{RN} = 0.08$$

and

$$\theta_{RMm} = 1.02, \sigma\theta_{RM} = 0.08$$

for the shafts of braced and bracing piers, respectively. The standard deviation

$$\sigma\theta_R = 0.06 - 0.08$$

is recommended by Szerszen *et al.* (2005) as statistical parameter for reliability analysis of eccentrically loaded columns.

4.2. Survival probabilities

For the sake of simplified but fairly exact probabilistic analysis of pier shafts, it is expedient to present Eqs (47) and (48), respectively in the form:

$$Z_N = R_{CN} - N_C \text{ or } Z_M = R_{CM} - M_C, \quad (49)$$

where

$$R_{CN} = \theta_{RN}R_N - \theta_N N_G \text{ or } R_{CM} = \theta_{RM}R_M - \theta_M M_G \quad (50)$$

are the conventional resistances of shafts and

$$N_C = \theta_N N_Q \text{ or } M_C = \theta_M M_Q \quad (51)$$

are the action effects caused by live loads of road bridges. The parameters R_{CN} and N_C, R_{CM} and M_C of the safety margins by Eqs (49) are statistically independent variables. The statistics of components of these safety margins may be expressed as:

$$R_{CNm} = \theta_{RNm}R_{Nm} - \theta_{Nm}N_{Gm}, \quad (52)$$

$$\sigma^2 R_{CN} = \theta_{RNm}^2 \sigma^2 R_N + R_{Nm}^2 \sigma^2 \theta_{RN} + \theta_{Nm}^2 \sigma^2 N_G + N_{Gm}^2 \sigma^2 \theta_N, \quad (53)$$

$$N_{Cm} = \theta_{Nm}N_{Qm}, \quad (54)$$

$$\sigma^2 N_C = \theta_{Nm}^2 \sigma^2 N_Q + N_{Qm}^2 \sigma^2 \theta_N, \quad (55)$$

$$R_{CMm} = \theta_{RMm}R_{Mm} - \theta_{Mm}M_{Gm}, \quad (56)$$

$$\sigma^2 R_{CM} = \theta_{RMm}^2 \sigma^2 R_M + R_{Mm}^2 \sigma^2 \theta_{RM} + \theta_{Mm}^2 \sigma^2 M_G + M_{Gm}^2 \sigma^2 \theta_M, \quad (57)$$

$$M_{Cm} = \theta_{Mm}M_{Qm} = \theta_{Mm}(N_{Qm}e_m + Q_{lm}h), \quad (58)$$

$$\sigma^2 M_C = \sigma^2\left(\theta_M M_Q\right) = \left(\theta_{Mm}N_{Qm}\right)^2 \sigma^2 e + \left(\theta_{Mm}e_m\right)^2 \sigma^2 N_Q + \left(\theta_{Mm}h\right)^2 \sigma^2 Q_l + 2\theta_{Mm}^2 e_m h \times \sigma N_Q \sigma Q_l + \left(N_{Qm}e_m + Q_{lm}h\right)^2 \sigma^2 \theta_M. \quad (59)$$

The survival probabilities of pier shafts may be calculated by the Eqs:

$$\mathbf{P}_{SN1} = \mathbf{P}(\beta_{N1}) = \int_0^\infty f_{R_{CN}}(x) F_{N_C}(x)\, dx, \quad (60)$$

$$\mathbf{P}_{SM1} = \mathbf{P}(\beta_{M1}) = \int_0^\infty f_{R_{CM}}(x) F_{M_C}(x)\, dx, \quad (61)$$

where $f_{R_{CN}}$, $f_{R_{CM}}$ are the density functions of normally distributed variables R_{CN} and R_{CM} given by Eq (50) and F_{N_C}, F_{M_C} are the cumulative distribution functions of lognormally distributed live action effects N_C and M_C expressed by Eq (51).

5. Numerical example

5.1. The parameters of design

The precast spun concrete shafts of annular cross-sections of braced piers are subjected to permanent, G_1, sustained, G_2, and live, Q, loads. The characteristic values of gravity compressive forces are presented as:

$$N_{Gk} = N_{G1k} + N_{G2k} = 1.08 + 0.38 = 1.46 \text{ (MN)}$$

and

$$N_{Qk} = 1.82 \text{ MN}.$$

According to *Eurocode* recommendations, the partial safety factors are:

$$\gamma_F = \gamma_G = \gamma_Q = 1.35,$$

$$\gamma_C = 1.5,$$

$$\gamma_S = 1.15.$$

The multiplication factor for actions

$$K_{F1} = 1.0$$

(for medium consequences of failure).

The statistics and design values of forces are as follows:

$$N_{Gm} = 1.46 \text{ MN},$$

$$\sigma^2 N_G = (0.1 N_{G1m})^2 + (0.25 N_{G2m})^2 = 0.0207 \text{ (MN)}^2,$$

$$N_{Qm} = \frac{N_{Qk}}{1+\gamma_{0.95}\delta Q} = \frac{1.82}{1+1.82\times 0.25} = 1.25 \text{ (MN)},$$

$$\sigma^2 N_Q = (\delta Q \times N_{Qm})^2 = (0.25 \times 1.25)^2 = 0.0977 \text{ (MN)}^2,$$

$$N_{Em} = N_{Gk} + N_{Qm} = 1.46 + 1.25 = 2.71 \text{ (MN)},$$

$$\sigma^2 N_E = 0.0207 + 0.0977 = 0.1184 \text{ (MN)}^2,$$

$$N_{Gd} = 1.35 \times 1.46 = 1.971 \text{ (MN)},$$

$$N_{Qd} = 1.35 \times 1.82 = 2.457 \text{ (MN)},$$

$$N_{Ed} = 1.971 + 2.457 = 4.428 \text{ (MN)}.$$

The parameters of pier shafts are as follows:

$$h = l_0 = 6.1 \text{ m},$$

$$\delta l_0 \approx 0.1,$$

$$\sigma^2 l_0 = (0.1 \times 6.1)^2 = 0.372 \text{ m}^2,$$

$$r_2 = 0.30 \text{ m},$$

$$r_1 = 0.20 \text{ m},$$

$$r_s = 0.25 \text{ m},$$

$$A_s = 0.00502 \text{ m}^2 \text{ (16}\varnothing\text{20 S800)},$$

$$A_{cm} = 0.152 \text{ m}^2,$$

$$\rho_m = \frac{A_s}{A_{cm}} = 0.033,$$

$$I = I_m = \frac{\pi\left(0.30^4 - 0.20^4\right)}{4} = 0.005105 \text{ m}^4,$$

$$\delta A_C = \delta I = \frac{1.2 - r_2}{150\left(r_2 - r_1\right)} = 0.0633,$$

$$\sigma^2 A_c = 92.6 \times 10^{-6} \text{ m}^4,$$

$$\sigma^2 I = 0.104 \times 10^{-6} \text{ m}^8,$$

$$\sigma^2 \rho = \frac{A_S}{A_{cm}^2}\sigma^2 A_C = 4.37 \times 10^{-6}.$$

According to Eq (12), the 1st order eccentricity

$$e_0 = \frac{6.1}{400} + \frac{0.30}{15} = 0.03525 \text{ (m)}.$$

The parameters of the concrete C50/60 are given by:

$$f_{ck} = 50 \text{ MPa},$$

$$f_{cm} = 58 \text{ MPa},$$

$$\alpha_{ccm} = 1 - 0.1\frac{1.46}{2.71} = 0.946,$$

$$k_{2m} = 0.85 - 1.7 \times 0.033 = 0.794,$$

$$f_{ccm} = 0.946 \times 0.794 \times 58 = 43.56 \text{ (MPa)},$$

$$\delta f_{cc} = 0.16,$$

$$\sigma^2 f_{cc} = (0.16 \times 43.56)^2 = 48.57 \text{ (MPa)}^2,$$

$$E_{cm} = 20(0.1 \times f_{cm})^{0.3} = 33.89 \text{ (GPa)},$$

$$\sigma^2 E_c = (0.15 \times 33.89)^2 = 25.84 \text{ (GPa)}^2,$$

$$E_{cd} = \frac{33.89}{1.2} = 28.24 \text{ (GPa)},$$

$$\alpha_{ccd} = 1 - 0.1\frac{1.971}{4.428} = 0.9555,$$

$$f_{ccd} = \frac{\alpha_{ccd} k_{2m} f_{ck}}{\gamma_c} = 25.29 \text{ (MPa)},$$

$$\Phi = 1.5.$$

The parameters of reinforcing high-strength bars are defined as follows:

$$f_{0.2k} = 800 \text{ MPa},$$

$$\sigma'_{scm} = \sigma'_{sck} = 452\left(1.18 + 4 \times 0.033\right) = 593 \text{ (MPa)},$$

$$\sigma'_{scd} = \frac{593}{1.15} = 515.7 \text{ (MPa)},$$

$$s^2 \sigma'_{sc} = \left(0.105 \times 593\right)^2 = 3877 \text{ (MPa)}^2.$$

The statistics of additional random variables are:

$$\theta_{Nm} = 1.0,$$

$$\sigma\theta_N = 0.10$$

and

$$\theta_{Rm} = 0.99,$$

$$\sigma\theta_R = 0.08.$$

5.2. Limit state design

The design value of buckling load for pier shafts by Eq (14) is defined as:

$$N_{Bd} = \pi^2 \frac{0.3}{1 + 0.5 \times 1.5 \frac{1.971}{4.428}} \times$$

$$28240 \frac{0.005105}{6.1^2} = 8.60 \text{ (MN)}.$$

According to Eq (18), the 2nd order eccentricity is:

$$e_d = 0.03525 \frac{8.60 + \left(\frac{\pi^2}{8} - 1\right)4.428}{8.60 - 4.428} = 0.0812 \text{ (m)}.$$

In this case, the design values of shaft response factors by Eqs (33) and (34) are expressed as follows:

$$k_{cd} = 1 - 0.3\frac{0.0812}{0.25\times(1+10\times 0.033)} = 0.927,$$

$$k_{sd} = 1 - 0.34\frac{0.0812}{0.25} = 0.890.$$

According to Eq (36), the design resisting force of shafts is presented as:

$$N_{Rd} = (0.927\times 0.152\times 25.29 +$$
$$0.890\times 0.00502\times 515.7)\frac{0.25}{0.0815+0.25} =$$
$$4.423\ (\mathrm{MN}) \approx N_{Ed} (= 4.428\ (\mathrm{MN})).$$

It shows that the analysed precast spun concrete shafts are suitable for bridge braced piers. However, when the consequences of failure may be high (K_{F1} = 1.1), the compressive force N_{ed} = 4.674 MN is inadmissibly more than N_{Rd} = 4.354 MN.

5.3. Probability-based design

According to Eq (8), the stiffness factor

$$K_{cm} = \frac{0.3}{1+0.5\times 1.5\frac{1.46}{2.71}} = 0.214.$$

Therefore, the statistics expressed by Eqs (15) and (16) are presented as:

$$N_{Bm} \approx \pi^2 0.214\times 33890\frac{0.005105}{6.1^2} = 9.806\ (\mathrm{MN}),$$

$$\sigma^2 N_B \approx 0.0837\times 10^{-6}\times 25.84\times 10^6 +$$
$$3.69\times 10^6\times 0.104\times 10^{-6} +$$
$$10.33\times 0.372 + 2105\times 71.6\ 10^{-6} = 6.543\ (\mathrm{MN})^2.$$

The statistics of 2nd order eccentricity by Eqs (19) and (20) are:

$$e_m = 0.03525\frac{9.806+0.2337\times 2.71}{9.806-2.71} = 0.0519\ (\mathrm{m}),$$

$$\sigma^2 e = \left[\frac{2.71\times 0.03525\times 1.2337}{(9.806-2.71)^2}\right]^2 6.543 +$$
$$\left[\frac{9.806\times 0.03525\times 1.2337}{(9.806-2.71)^2}\right]^2 0.1184 = 44.33\times 10^{-6}\ (\mathrm{m}^2).$$

The mean values of response factors by Eqs (33) and (34) are:

$$k_{cm} = 1 - 0.3\frac{0.0519}{0.25\times 1.33} = 0.953,$$

$$k_{sm} = 1 - 0.34\frac{0.0519}{0.25} = 0.929.$$

Then, according to Eqs (37) and (38), the statistics of resistance $R_N = N_R$ are:

$$R_{Nm} = (0.953\times 0.152\times 43.56 + 0.929\times$$
$$0.00502\times 593)\frac{0.25}{0.0519+0.25} = 7.517\ (\mathrm{MN}),$$

$$\sigma^2 R_N = 0.0144\times 48.57 + 1182\times 92.6\times 10^{-6} +$$
$$14.93\times 10^{-6}\times 3877 + 1102\times 43.5\times 10^{-6} = 0.914\ (\mathrm{MN})^2.$$

According to Eqs (52), (53) and (54), (55), the means and variances of conventional resistance, R_{NC}, and destroying variable live force, N_C, are:

$$R_{NCm} = 0.99\times 7.517 - 1.0\times 1.46 = 5.982\ (\mathrm{MN}),$$

$$\sigma^2 R_{NC} \approx 0.99^2\times 0.914 + 7.517^2\times 0.0064 +$$
$$1.0^2\times 0.0207 + 1.46^2\times 0.01 = 1.30\ (\mathrm{MN})^2,$$

$$N_{Cm} = 1.0\times 1.25 = 1.25\ (\mathrm{MN}),$$

$$\sigma^2 N_C = 1.0^2\times 0.0977 + 1.25^2\times 0.01 = 0.1133\ (\mathrm{MN})^2.$$

Thus, according to Eqs (60) and (46), the survival probability and reliability index of pier shafts are equal to

$$\mathbf{P}_{SN1} = 0.999954$$

and

$$\beta_{N1} = 3.91.$$

It shows that their structural safety is sufficient and constructive solution is effective, i.e.

$$\beta_{N1} \approx \beta_T (= 4.0).$$

When the consequences of failure may be high and δQ = 0.30, the reliability index of analysed piers β_{N1} = 3.79 may be to small.

5.4. On acceptability of probability distribution laws

The data given in Tables 1 and 2 show, that all design variables R_N, N_G and N_Q or R_M, M_G and M_Q cannot be adequately modelled only by the normal or lognormal distribution laws. It may lead designers to an overestimation of predicted reliability indices of shafts. As it is shown, case 1 of probability distribution laws for the components of safety margins of considered pier shafts corroborated its acceptability in probability-based analysis of bridge piers.

6. Conclusions

The design features of economically reasonable precast spun concrete pier shafts depend on constructional solution of braced and bracing piers of road bridges and footbridges. The analysis of load-carrying capacity and structural reliability of pier shafts of annular cross-sections reinforced by steel bars uniformly distributed throughout their perimeter may be determined by unsophisticated semi-probabilistic and probability-based concepts and approaches demonstrated in this paper. They may stimulate engineers having min appropriate skills to use full probabilistic approaches in design practice more courageously.

Table 1. The reliability indices for the shaft of a braced pier

Forces	Mean, MN	Variance, $(MN)^2$	Case 1		Case 2		Case 3	
			Distribution	β_{N1}	Distribution	β_{N2}	Distribution	β_{N3}
$\theta_R N_R = \theta_R R_N$	7.442	1.2578	Normal	3.91	Normal	3.98	Lognormal	4.85
$\theta_N N_G$	1.460	0.0420	Normal		Normal		Lognormal	
$\theta_N N_Q$	1.250	0.1133	Lognormal		Normal		Lognormal	

Table 2. The reliability indices for the shaft of a bracing pier (Kudzys and Kliukas, 2008)

Forces	Mean, MN	Variance, $(MN)^2$	Case 1		Case 2		Case 3	
			Distribution	β_{M1}	Distribution	β_{M2}	Distribution	β_{M3}
$\theta_R M_R = \theta_R M_N$	9.492	0.9119	Normal	3.93	Normal	4.88	Lognormal	4.08
$\theta_M M_G$	0.318	0.0030	Normal		Normal		Lognormal	
$\theta_M M_Q$	3.034	0.6680	Lognormal		Normal		Lognormal	

For shafts of annular cross-sections of braced and bracing piers, the 2nd order eccentricities of destroying forces may be expressed in a simple and easily perceptible manner. It is expedient to analyse the eccentrically loaded spun concrete shafts of braced and bracing bridge piers as the columns under compression with a bending moment and under bending with a compression force, respectively.

The objective assessment of structural safety level for bridge pier shafts may be introduced only by reliability index β using proper distributions for the components of their safety margins. The data presented in this paper corroborated that a lognormal distribution is to be used for variable live actions and a Gaussian distribution may be used for the joint values of permanent effects and shaft resistances. For spun concrete shafts of piers, the target reliability index β_T may be selected equal to 4.0, as it is recommended by the investigators in the USA for eccentrically loaded reinforced concrete columns.

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